A MULTIDIMENSIONAL ENERGY OPERATOR FOR IMAGE PROCESSING

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Abstract

The 1-D nonlinear differential operator $\Psi(f) = (f')^2 - ff''$ has been recently introduced to signal processing and has been found very useful for estimating the parameters of sinusoids and the modulating signals of AM–FM signals. It is called an *energy operator* because it can track the energy of an oscillator source generating a sinusoidal signal. In this paper we introduce the multidimensional extension $\Phi(f) = ||\nabla f||^2 - f\nabla^2 f$ of the 1-D energy operator and briefly outline some of its applications to image processing. We discuss some interesting properties of the multidimensional operator and develop demodulation algorithms to estimate the amplitude envelope and instantaneous frequencies of 2-D spatially-varying AM–FM signals, which can model image textures. The attractive features of the multidimensional operator operator and the related amplitude/frequency demodulation algorithms are their simplicity, efficiency, and ability to track instantaneously-varying spatial modulation patterns.

1 Introduction

In his work on nonlinear modeling of speech production [15, 16], Teager developed a nonlinear differential operator Ψ_c for 1-D continuous-time signals $f(t)$, defined as

$$\Psi_c(f)(t) \triangleq [f'(t)]^2 - f(t)f''(t) \tag{1}$$

where $f'() = df/dt$ and $f''() = d^2f/dt^2$. The discrete-time counterpart of Ψ_c is the operator

$$\Psi_d(f)(n) \triangleq f^2(n) - f(n-1)f(n+1) \tag{2}$$

for discrete-time signals $f(n)$. Both operators were first introduced systematically by Kaiser [4, 5]. Ψ_c is an 'energy-tracking' operator because it can track the energy of simple harmonic oscillators that produce sinusoidal oscillatory signals; this energy is proportional to both the amplitude squared and the frequency squared of the oscillation. Hence we shall refer to Ψ_c and Ψ_d as the 1-D *energy operators.* Kaiser [5] found the following properties of Ψ_c: for any constants A, r, and ω_0 and for any signals f and g

$$\Psi_c[Ar^t\cos(\omega_0 t + \theta)] = A^2 r^{2t}\omega_0^2 \tag{3}$$

$$\Psi_c(fg) = f^2\Psi_c(g) + g^2\Psi_c(f) \tag{4}$$

The discrete operator also has similar properties [4]:

$$\Psi_d[Ar^n\cos(\Omega_0 n + \theta)] = A^2 r^{2n}\sin^2(\Omega_0) \tag{5}$$

$$\Psi_d(fg) = f^2\Psi_d(g) + g^2\Psi_d(f) - \Psi_d(f)\Psi_d(g) \tag{6}$$

The energy operators are very efficient in instantaneously estimating the modulating signals of 1-D AM–FM signals. Specifically, Maragos, Kaiser, and Quatieri [8, 10] have shown that the energy operators can approximately estimate the envelope of signals with amplitude modulation (AM) and the instantaneous frequency of signals with frequency modulation (FM). For 1-D AM–FM signals

$$f(t) = a(t)\cos[\phi(t)] \tag{7}$$

that have a combined AM and FM structure, they have also found that the energy operator tracks the energy product

$$\Psi[a(t)\cos(\phi(t))] \approx a^2(t)\omega_i^2(t) \tag{8}$$

where $\omega_i(t) = d\phi(t)/dt$ is the instantaneous (angular) frequency. This approximate result is valid (i.e., the approximation error is negligible) if the time-varying amplitude $a(t)$ and frequency $\omega_i(t)$ do not vary too fast in time or too greatly compared with the carrier. Sufficient conditions for this are a small amount of modulation and the amplitude and frequency modulating signals to be bandlimited with bandwidths much smaller than the carrier frequency [8]. Further, by applying Ψ_c to the derivative $f'(t)$ and combining the energy output with (7) they also developed an *energy separation algorithm (ESA)* [9, 10] that separates the energy product (7) into amplitude and frequency components. Thus the ESA fully demodulates the AM–FM signal by estimating its amplitude envelope $|a(t)|$ and instantaneous frequency $\omega_i(t)$. Similar results and algorithms have been derived for discrete-time AM–FM signals [8, 10].

In addition to the great promise of the energy operator and the ESA for communications applications due to their efficiency for AM–FM demodulation, their major application so far has been the instantaneous tracking of modulations in speech resonances. Motivated by several nonlinear fluid dynamic phenomena during speech production, Maragos, Quatieri, and Kaiser [6, 7] have modeled speech resonances with AM–FM signals. Another application of the energy operator is as an 'event detector'. For example, Quatieri, Kaiser, and Maragos [13] applied the energy operator to detect transient signal signatures in the presence of an AM–FM noise background. The effects of noise on the performance of the energy operator and the ESA have been studied in detail by Bovik, Maragos, and Quatieri [2].

In this paper we introduce a multidimensional extension Φ_c of Ψ_c. First we derive many useful properties of Φ_c and show how it can be used to estimate the parameters of multidimensional sinusoids. Then we extend the analysis to multidimensional AM–FM signals, i.e., multidimensional sinusoids with spatially-varying[1] amplitude envelope and instantaneous frequencies. Specifically, we derive an algorithm (extension of the 1-D ESA) that can demodulate a multidimensional AM–FM signal and estimate its envelope and instantaneous frequencies. Bovik et al. [1] have demonstrated that such 2-D AM–FM signals can model well image textures. Therefore, our AM–FM demodulation algorithms based on the multidimensional energy operator are very promising for image texture modeling since they provide a simple and effective way to estimate the model parameters.

For 2-D signals, if we replace the partial derivatives in Φ_c with one-sample differences we obtain a discrete-domain 2-D energy operator Φ_d, which is identical to the one developed in Yu, Mitra, and Kaiser [17] for digital image edge detection, and also used in [11] for image enhancement. We repeat the theoretical analysis for the discrete operator and derive a related discrete AM–FM demodulation algorithm. The energy operators are then generalized to vector-valued signals using a similar framework as in the multidimensional case. Finally, we briefly outline some of the applications of multidimensional energy operators to image processing.

[1] Although we refer to a multidimensional signal argument as 'spatial', our discussion applies not only to images but arbitrary signals.

2 Continuous-domain Multidimensional Energy Operator

Let $f(\vec{x})$ be a ν-D real-valued signal with a continuous argument $\vec{x} = (x_1, ..., x_\nu) \in \mathbf{R}^\nu$, $\nu = 2, 3, ...$ Then we define the ν-D energy operator by

$$\Phi_c(f)(\vec{x}) \triangleq ||\nabla f(\vec{x})||^2 - f(\vec{x})\nabla^2 f(\vec{x}) \tag{9}$$

where

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, ..., \frac{\partial f}{\partial x_\nu}\right) \tag{10}$$

is the gradient of f,

$$||\nabla f||^2 = \left(\frac{\partial f}{\partial x_1}\right)^2 + ... + \left(\frac{\partial f}{\partial x_\nu}\right)^2 \tag{11}$$

is the Euclidean norm squared of the gradient, and

$$\nabla^2 f = \sum_{k=1}^{\nu} \frac{\partial^2 f}{\partial x_k^2} \tag{12}$$

is the Laplacian of f. From its definition it follows directly that we can express $\Phi_c(f)$ as

$$\Phi_c(f) = \sum_{k=1}^{\nu} \left(\frac{\partial f}{\partial x_k}\right)^2 - f\left(\frac{\partial^2 f}{\partial x_k^2}\right) = \sum_{k=1}^{\nu} \Psi_{c,k}(f) \tag{13}$$

where

$$\Psi_{c,k}(f) \triangleq \left(\frac{\partial f}{\partial x_k}\right)^2 - f\,\frac{\partial^2 f}{\partial x_k^2} \tag{14}$$

Thus the output of the Φ_c is a *sum of 'energy components'*. Each energy component is the output of the 1-D energy operator Ψ_c applied along each one of the ν directions x_k. Hence, in analogy with the 1-D case, we shall refer to Φ_c as the *'multidimensional energy operator'*. Next we derive a few of its properties.

Let $f(\vec{x})$ and $g(\vec{x})$ be two ν-D signals. Applying Φ_c to their product yields a similar result as in the 1-D case. Specifically, since in general

$$\nabla(fg) = g\nabla f + f\nabla g \tag{15}$$

$$\nabla^2(fg) = g\nabla^2 f + f\nabla^2 g + 2(\nabla f)\cdot(\nabla g) \tag{16}$$

where '$\cdot$' denotes inner product, it follows that

$$\Phi_c(fg) = f^2\Phi_c(g) + g^2\Phi_c(f) \tag{17}$$

For a multidimensional exponential signal, the output of the energy operator is identically zero:

$$\Phi_c[\exp\left(\sum_{k=1}^{\nu} c_k x_k\right)] = 0 \tag{18}$$

where c_k are arbitary constants. Combining this with (17) implies that we can extend all the results in this paper to signals that contain an arbitrary constant scaling factor A and/or an exponential multiplicative component because

$$\Phi_c[A\exp\left(\sum_{k=1}^{\nu} c_k x_k\right) f(\vec{x})] = A^2 \exp\left(2\sum_{k=1}^{\nu} c_k x_k\right) \Phi_c[f(\vec{x})] \tag{19}$$

2.1 Cosines with Constant Amplitude/Frequencies

Applying Φ_c to a ν-D cosine

$$f(\vec{x}) = A\cos(\vec{\omega}_c \cdot \vec{x} + \theta) \tag{20}$$

with constant phase offset θ, constant amplitude A, and *constant* frequency vector

$$\vec{\omega}_c = (\omega_{c,1}, ..., \omega_{c,\nu}) \tag{21}$$

yields

$$\Phi_c[A\cos(\vec{\omega}_c \cdot \vec{x} + \theta)] = A^2 \left(\sum_{k=1}^{\nu} \omega_{c,k}^2 \right) = A^2 ||\vec{\omega}_c||^2 \tag{22}$$

Thus, when Φ_c is applied to a multidimensional cosine, it yields the product of the amplitude squared and the frequency vector norm squared.

Now to estimate the individual $\nu + 1$ parameters $|A|, \omega_{c,1}, ..., \omega_{c,\nu}$ we also apply Φ_c to the ν partial derivatives

$$f_k(\vec{x}) = \frac{\partial[A\cos(\vec{\omega}_c \cdot \vec{x} + \theta)]}{\partial x_k} = -A\omega_{c,k}\sin(\vec{\omega}_c \cdot \vec{x} + \theta) \tag{23}$$

of the cosine f. Then, by (22),

$$\Phi_c(f_k)(\vec{x}) = (A\omega_{c,k})^2 ||\vec{\omega}_c||^2 \tag{24}$$

for all $k = 1, ..., \nu$. By combining (22) and (24) we obtain the following $\nu + 1$ equations for exact estimation of the absolute amplitude and ν frequencies:

$$\omega_{c,1} = \sqrt{\frac{\Phi_c\left(\frac{\partial f}{\partial x_1}\right)}{\Phi_c(f)}} \tag{25}$$

$$\vdots \quad \vdots \quad \vdots \tag{26}$$

$$\omega_{c,\nu} = \sqrt{\frac{\Phi_c\left(\frac{\partial f}{\partial x_\nu}\right)}{\Phi_c(f)}} \tag{27}$$

$$|A| = \frac{\Phi_c(f)}{\sqrt{\sum_{k=1}^{\nu} \Phi_c\left(\frac{\partial f}{\partial x_k}\right)}} \tag{28}$$

We call the above equations the multidimensional *continuous energy separation algorithm (CESA)*. They are an extension of the 1-D CESA developed in [9, 10].

2.2 AM–FM Signals

Consider the real-valued ν-D AM–FM signal

$$f(\vec{x}) = a(\vec{x})\cos[\phi(\vec{x})] \tag{29}$$

where $a(\vec{x})$ is a spatially-varying amplitude, $\phi(\vec{x})$ is the phase signal,

$$\vec{\omega}(\vec{x}) \triangleq \nabla\phi(\vec{x}) = (\omega_1(\vec{x}), ..., \omega_\nu(\vec{x})) \tag{30}$$

is the spatially-varying ν-D *instantaneous frequency vector*, and $\omega_k(\vec{x})$ is the k-th instantaneous angular frequency signal (in radians/cycle). Assuming for each k that ω_k is non-negative, we can always express it as

$$\omega_k(\vec{x}) = \omega_{c,k} + \omega_{m,k} q_k(\vec{x}) \tag{31}$$

where $\omega_{c,k}$ is a constant center frequency, $q_k(\vec{x}) \in [-1,1]$ is the k-th frequency modulating signal, and $\omega_{m,k}$ is the maximum deviation of ω_k from its center value. Henceforth we assume that $0 \leq \omega_{m,k} \leq \omega_{c,k}$.

Applying Φ_c to f yields

$$\Phi_c[a\cos(\phi)] = a^2||\vec{\omega}||^2 - \frac{1}{2}a^2\sin(2\phi)\nabla^2\phi + \cos^2(\phi)\Phi_c(a) \tag{32}$$

For demodulation the desired term in (32) is $a^2||\vec{\omega}||^2$. We view the rest of the terms as approximation error and show next that they are negligible under realistic assumptions.

Assume that $a(\vec{x})$ is bandlimited in a circular frequency sphere of radius ω_a. Namely, if $A(\vec{u})$ is its ν-D Fourier transform, then $A(\vec{u}) = 0$ for $||\vec{u}|| > \omega_a$. Then if we define the mean spectral absolute value of a as

$$\mu_a = \frac{1}{(2\pi)^\nu} \int_{-\omega_a}^{\omega_a} \cdots \int_{-\omega_a}^{\omega_a} |A(\vec{u})| du_1 \ldots du_\nu \tag{33}$$

it can be shown that for each k

$$\begin{aligned} |a(\vec{x})| &\leq a_{max} \leq \mu_a && (34) \\ \left|\frac{\partial a}{\partial x_k}\right| &\leq \omega_a \mu_a && (35) \\ \left|\frac{\partial^2 a}{\partial x_k^2}\right| &\leq \omega_a^2 \mu_a && (36) \\ |\Phi_c(a)(\vec{x})| &\leq 2\omega_a^2 \mu_a^2 && (37) \end{aligned}$$

where $a_{max} = \sup_{\vec{x}} |a(\vec{x})|$. Assume also that each frequency signal $\omega_k(\vec{x})$ is bandlimited with bandwidth $\omega_{f,k} < \omega_{c,k}$. Then we can consider the approximation

$$\Phi_c[a\cos(\phi)] \approx a^2||\vec{\omega}||^2 \tag{38}$$

with an approximation error

$$E(\vec{x}) = \Phi_c[a\cos(\phi)] - a^2||\vec{\omega}||^2 \tag{39}$$

that is bounded by

$$|E(\vec{x})| \leq \left(2\omega_a^2 + + \frac{1}{2}\sum_{k=1}^{\nu} \omega_{m,k}\omega_{f,k}\mu_{q_k}\right)\mu_a^2 \tag{40}$$

since

$$\frac{\partial^2 \phi}{\partial x_k^2} = \omega_{m,k}\frac{\partial q_k}{\partial x_k} \tag{41}$$

Assuming that $a_{max} \approx \mu_a$ (which is true with equality if a is a cosine or has linear Fourier phase), the realistic conditions

$$\omega_a \ll \min_k \omega_{c,k} \quad \text{and} \quad \omega_{m,k}\omega_{f,k} \ll (\omega_{c,k})^2 \quad \forall k \tag{42}$$

make the maximum absolute value of the error E much smaller than the maximum absolute value of the desired term. Thus, under such conditions, the approximation (38) is valid in the sense that the relative error is $\ll 1$. Note that conditions (42) imply that the amplitude and frequency signals do not vary too fast in space or too greatly compared with the carriers.

Now let us apply Φ_c to the partial derivatives

$$\frac{\partial f}{\partial x_k} = \frac{\partial a}{\partial x_k}\cos(\phi) - a\omega_k \sin(\phi) \tag{43}$$

Due to (42) the second term in $\partial a/\partial x_k$ has a much larger order of magnitude of its maximum absolute value compared with the first term. Thus we approximate $\partial f/\partial x_k \approx -a\omega_k \sin(\phi)$ and apply (38) to obtain

$$\Phi_c\left(\frac{\partial f}{\partial x_k}\right) \approx \Phi_c[a\omega_k \sin(\phi)] \approx a^2\omega_k^2||\vec{\omega}||^2 \tag{44}$$

for each k. Combining this equation and (38) yields the following CESA

$$\sqrt{\frac{\Phi_c\left(\frac{\partial f}{\partial x_1}\right)}{\Phi_c(f)}} \approx \omega_1(\vec{x}) \tag{45}$$

$$\vdots \quad \vdots \quad \vdots \tag{46}$$

$$\sqrt{\frac{\Phi_c\left(\frac{\partial f}{\partial x_\nu}\right)}{\Phi_c(f)}} \approx \omega_\nu(\vec{x}) \tag{47}$$

$$\frac{\Phi_c(f)}{\sqrt{\sum_{k=1}^{\nu}\Phi_c\left(\frac{\partial f}{\partial x_k}\right)}} \approx |a(\vec{x})| \tag{48}$$

This algorithm can estimate at each location $\vec{x}$ the amplitude envelope and instantaneous frequencies of the spatially-varying AM–FM signal.

3 Discrete-space Energy Operator for Image Signals

In general, if we replace derivatives in Φ_c with one-sample differences we obtain a discrete-space energy operator. For notational simplicity, we restrict our discussion to 2-D signals, e.g., still images.

The alternative interpretation (13) of Φ_c as a sum of energy components along different directions allows us to extend it to discrete-space signals $f(m,n)$. Specifically, replacing each of these energy components with outputs from 1-D discrete-time energy operators Ψ_d yields the 2-D discrete-space energy operator

$$\Phi_d(f)(m,n) = \Psi_{d,1}(f)(m,n) + \Psi_{d,2}(f)(m,n) \tag{49}$$

$$\triangleq 2f^2(m,n) - f(m-1,n)f(m+1,n) - f(m,n-1)f(m,n+1) \tag{50}$$

where

$$\Psi_{d,1}(f)(m,n) \triangleq f^2(m,n) - f(m-1,n)f(m+1,n) \tag{51}$$

applies horizontally the 1-D energy operator on all rows of f, whereas $\Psi_{d,2}$ operates on the columns. The expression (50) is identical to the discrete operator developed in [17] for digital image edge detection.

Applying Φ_d to a 2-D sinusoid with constant amplitude/frequencies yields

$$\Phi_d[A\cos(\Omega_1 m + \Omega_2 n + \theta)] = A^2[\sin^2(\Omega_1) + \sin^2(\Omega_2)] \tag{52}$$

Consider now a discrete AM–FM signal

$$f(m,n) = a(m,n)\cos[\phi(m,n)] \tag{53}$$

Its horizontal instantaneous frequency (in radians/sample)

$$\Omega_1(m,n) \triangleq \frac{\partial \phi}{\partial m} \Omega_{c,1} + \Omega_{m,1} q_1(m,n) \tag{54}$$

has center frequency $\Omega_{c,1}$ and maximum frequency deviation $\Omega_{m,1} \leq \Omega_{c,1}$. The frequency modulating signal $q_1(m,n)$ is assumed to be a mathematical function with a known computable integral. Likewise for the vertical frequency signal Ω_2. All discrete-space frquencies are assumed to be in $[0,\pi]$. We henceforth assume that a is bandlimited with bandwidth Ω_a and that both frequency signals are finite weighted sums of sinusoids[2] and bandlimited with bandwidth Ω_f. Then under the realistic assumptions

$$\Omega_a \ll \min_k \Omega_{c,k}, \quad \Omega_f \ll 1, \quad \Omega_{m,k} \ll \Omega_{c,k} \tag{55}$$

it follows from (49) and by working as in the 1-D case in [8, 9] that

$$\Phi_d[a(m,n)\cos(\phi(m,n)] \approx a^2(m,n)(\sin^2[\Omega_1(m,n)] + \sin^2[\Omega_2(m,n)]) \tag{56}$$

Now replacing the partial derivatives of the previous section with *symmetric* 3-sample differences in each direction yields the 2-d signals

$$\begin{aligned} g_1(m,n) &= [f(m+1,n) - f(m-1,n)]/2 & (57) \\ g_2(m,n) &= [f(m,n+1) - f(m,n-1)]/2 & (58) \\ & & (59) \end{aligned}$$

which are 2-D AM–FM signals with amplitude and instantaneous frequencies that do not vary too fast or too much compared with the carriers $\Omega_{c,k}$. Hence (see also [9, 10] for the 1-D case)

$$\begin{aligned} \Phi_d[g_1(m,n)] &\approx a^2(m,n)\sin^2[\Omega_1(m,n)](\sin^2[\Omega_1(m,n)] + \sin^2[\Omega_2(m,n)]) & (60) \\ \Phi_d[g_2(m,n)] &\approx a^2(m,n)\sin^2[\Omega_2(m,n)](\sin^2[\Omega_1(m,n)] + \sin^2[\Omega_2(m,n)]) & (61) \end{aligned}$$

Combining (56),(60),(61) yields a *discrete energy separation algorithm (DESA)*

$$\begin{aligned} \arcsin\left(\sqrt{\frac{\Phi_d[f(m+1,n) - f(m-1,n)]}{4\Phi_d[f(m,n)]}}\right) &\approx \Omega_1(m,n) & (62) \\ \arcsin\left(\sqrt{\frac{\Phi_d[f(m,n+1) - f(m,n-1)]}{4\Phi_d[f(m,n)]}}\right) &\approx \Omega_2(m,n) & (63) \\ \frac{2\Phi_d[f(m,n)]}{\sqrt{\Phi_d[f(m+1,n) - f(m-1,n)] + \Phi_d[f(m,n+1) - f(m,n-1)]}} &\approx |a(m,n)| & (64) \end{aligned}$$

We call this algorithm *DESA–2* because it approximates derivatives with differences that are two samples apart. (It is also possible to derive an alternative DESA by using asymmetric one-sample differences as in [9, 10].) The DESA–2 can estimate at each location the amplitude enevelope and two instantaneous ferquency signals of a spatial AM–FM signal.

If the AM–FM signal has *constant* amplitude A and constant frequencies $\Omega_{c,1}$ and $\Omega_{c,2}$, then the DESA–2 equations provide an *exact* estimate of the amplitude $|a(m,n)| = |A|$ and frequencies $\Omega_1(m,n) = \Omega_{c,1}$ and $\Omega_2(m,n) = \Omega_{c,2}$.

[2]Our discussion and results also apply to the case where the frequency signals are linear ramps.

4 Energy Operators for Vector-Valued Signals

The framework of multidimensional energy operators is useful in extending the 1-D energy operators to vector-valued signals.

Consider a 1-D vector-valued signal

$$\vec{f}(t) = (f_1(t), f_2(t), ..., f_n(t)) \tag{65}$$

where all n components are real-valued. Define its vector derivative

$$\vec{f'} = (f_1', f_2', ..., f_n') \tag{66}$$

and its second derivative $\vec{f''} = (\vec{f'})'$. Then we define an energy operator for vector-values signals:

$$\Theta[f(t)] = ||f'(t)||^2 - f(t) \cdot f''(t) \tag{67}$$

It is simple to show that

$$\Theta(f) = \sum_{k=1}^{n} \Psi_c(f_k) \tag{68}$$

Hence, the energy of the vector-valued signal is the sum of the energies of its scalar component signals.

There are several applications of energy operators for vector-valued signals outlined next:

Complex signals: Let $f(t)$ a complex-valued signal. Bovik et al. [3] defined an energy operator for complex-valued signals f as

$$C(f)(t) \triangleq |f'(t)|^2 - \text{Real}[f^*(t) f''(t)] \tag{69}$$

where $()^*$ denotes complex conjugate. This complex operator has many interesting properties. Now by forming a vector-valued signals with $n = 2$, $f_1 = \text{Real}(f)$, and $f_2 = \text{Imag}(f)$, it follows that

$$C(f) = \Psi_c[\text{Real}(f)] + \Psi_c[\text{Imag}(f)] \tag{70}$$

Thus the analysis of applying energy operators to complex-valued signals can be reduced to simply analyzing separately their real and imaginary parts.

Array of uncoupled oscillators:. Let $f(t)$ be the position vector tracing a continuous smooth curve in n-D space. Then $f'(t)$ is the velocity vector tangent to the curve and $f''(t)$ is the acceleration vector. Then $||f'||^2$ is kinetic energy (per unit mass) whereas $-f \cdot f''$ is potential energy.

The energy operator Θ can be directly extended to 2-D vector-valued signals $f(x, y)$ by replacing the vector derivative f' with the matrix derivative $[\partial f_i / \partial x_j]$, the Euclidean vector with Frobenious matrix norm, and the vector 2nd derivative f'' with the vector Laplacian $(\nabla^2 f_1, ..., \nabla^2 f_n)$.

5 Discussion

Several types of image textures can be modeled by spatial modulation models of the AM–FM type with narrow-band (i.e., slowly-varying) amplitude and frequency signals, as demonstrated by Bovik et al. [1]. The amplitude signal $a(m, n)$ models intensity contrast variations, whereas the frequency signals Ω_1, Ω_2 convey information about the 'locally emergent frequencies'. In this paper we have shown that, if the amplitude and instantaneous frequency signals do not vary too fast in space or too much in value compared to their mean values, then we can use the 2-D energy operator and the 2-D DESA to estimate the parameters of these models. Given the importance and applicability of these AM–FM models the 2-D energy operator and 2-D DESA become important tools for their analysis. The advantages of the DESA is simplicity, efficiency, low computational complexity, instantaneous adaptation due to the differential nature of the energy operators, and ability to track spatial modulation patterns. in addition, the 2-D energy operator can be used as a spatial event detector. Finally, the 2-D energy operator for vector-valued signals has potential applications to vector-valued images, e.g., multispectral images.

Acknowledgements

P. Maragos was supported by the National Science Foundation under Grant MIP-91-20624 and by a National Science Foundation Presidential Young Investigator Award under Grant NSF MIP–86–58150 with matching funds from Xerox. A.C. Bovik was supported by a Faculty Research Assignment while on sabbatical at Harvard and by a grant from Texas Instruments, Inc. T.F. Quatieri was sponsored in part by the Dept. of the Air Force and in part by by the Naval Submarine Medical Research Laboratory.

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