OVERVIEW OF ADAPTIVE MORPHOLOGY: TRENDS AND PERSPECTIVES

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ABSTRACT

In this paper we briefly overview emerging trends in 'Adaptive Morphology', i.e. work related to the theory and/or applications of image analysis filters, systems, or algorithms based on mathematical morphology, that are adaptive w.r.t. to space or intensity or use any other adaptive scheme. We present a new classification of work in this area structured along several major theoretical perspectives. We then sample specific approaches that develop spatially-variant structuring elements or intensity level-adaptive operators, modeled and implemented either via conventional nonlinear digital filtering or via geometric PDEs. Finally, we discuss some applications.

Index Terms— Adaptive filters, Morphological image analysis

1. INTRODUCTION

In image and signal processing, adaptivity of a system or algorithm means its capability to automatically adjust its parameters to the input data aiming at optimizing some criterion. In the processing design, this adaptation should take into account the spatial, dynamical and/or temporal and range information which is available or can be computed from the data. The usefulness and necessity of adaptive algorithms are evident if one considers the variability of the signals or images that should commonly be processed by a unique algorithm, the internal variability of the data in a single image or image sequence, the a priori knowledge (e.g. context or noise) that needs to be incorporated in the processing, and the requirements of the processing in terms of resulting signal properties (for instance the preservation of certain image structures). We mention a few practical motivations: adapting to the luminance, or the contrast, or the gradient norm or the gradient direction of the image function at every point in space is fundamental if one wants to encourage intra-region smoothing while preserving edges; adapting to a possibly spatially-varying camera perspective is of crucial importance in many surveillance situations; the evolution of the image data throughout the time is also a precious information for video compression.

There are two fundamental questions when dealing with adaptive algorithms or adaptive transforms. First, how to mathematically define such operations? Second, how to practically design adaptive transforms, i.e., how to define the link between the transformation parameters and the image data? These questions have received an increasing interest in the image processing community, judging by the great numbers of publications that refer to adaptive algorithms.

In this paper, we briefly survey the state of the art on these questions in the field of mathematical morphology (MM) [12, 13, 26], which is a powerful nonlinear methodology for representing and analyzing geometrical structures in images and signals based on tools from set and lattice theory, topology and stochastic geometry, with numerous applications in image enhancement, feature extraction, multiscale filtering, detection and segmentation. We discuss three major perspectives and corresponding research directions for adaptive MM: (i) adaptivity w.r.t. the spatial neighborhood of morphological operators, (ii) algebraic principles such as group and representation theory to unify important aspects of the adaptive operators, and (iii) adaptivity w.r.t. how the operators process the image level sets at different levels. Our survey includes issues from the theoretical, design, computational and applications aspects of these directions. Our discussion of the modeling and implementation aspects of the adaptive operators in categories (i) and (ii) mainly focuses on the conventional filtering view, from which they appear as min-max combinations of nonlinear (sup/inf) spatially-variant convolutions, whereas in category (iii) we also add the viewpoint of partial differential equations (PDEs).

Due to the limited paper size, our references are limited and only indicative. More can be found in the the papers we cite.

2. THEORETICAL FRAMEWORKS FOR ADAPTIVE MM

Morphological operators, which include well-known rank, median and stack-type nonlinear filters, were originally defined so that they satisfy important properties. Translation invariance is a fundamental one. If $f(x)$ is a real image (or a function) defined on a space domain $\mathbb{E}$ such as $\mathbb{R}^d$ of $\mathbb{Z}^d$, a *translation-invariant (TI)* operator is an operator ψ such that for each input f and each (h, v) in $\mathbb{E} \times \mathbb{R}$

$$\psi(f_{h,v}) = [\psi(f)]_{h,v} \ , \quad f_{h,v}(x) \ := \ f(x-h) + v$$

The operator is called *horizontal-translation-invariant (HTI)* or *spatially-invariant* if it commutes only w.r.t. a horizontal (spatial) shift, and *vertical-translation-invariant (VTI)* if it commutes only w.r.t. a vertical (value) shift. If we consider only TI operators, then every signal dilation (every increasing operator that distributes with supremum $\bigvee$) δ and every erosion (every increasing operator that distributes with infimum $\bigwedge$) ε are Minkowski function additions $\oplus$ and subtractions $\ominus$; i.e., we can find a fixed function, called the *structuring element (SE)*, $g(x)$ such that $\delta(f) \ = \ f \oplus g$ and $\varepsilon(f) = f \ominus g$ where

$$f \oplus g(x) = \bigvee_{y \in B} f(x-y) + g(y), \quad f \ominus g(x) = \bigwedge_{y \in B} f(x+y) - g(y) \qquad (1)$$

and $B \subseteq \mathbb{E}$ is the support of $g(x)$. If g is flat, i.e. zero over its support, we obtain the *flat* dilation $f \oplus B$ and erosion $f \ominus B$ of f by B. Otherwise, (1) are the *weighted* dilation and erosion. More complex morphological operators/filters are formed by sup/inf superpositions and/or compositions of the dilations and erosions. The basic operations (1) are nonlinear convolutions, which represent the action of the combined filter as moving-window operations over the spatial domain $\mathbb{E}$. An alternative domain is the range, where the basic operators can be interpreted as operating on the level sets. For example, if all the involved SEs are sets, then the combined operators ψ are *flat*, i.e. they obey *threshold superposition*:

$$\psi(f) = \bigvee_{v \in \mathbb{R}} v \cdot \psi(\chi_v(f)) \tag{2}$$

where $\chi_v(f)$ is the indicator function of the *level set* $X_v(f) = \{x : f(x) \geq v\}$ of f.

For fixed SEs, the basic operators (1) and all their parallel and serial combinations are TI. However, when dealing with adaptive morphological operators, the TI property may be lost if the action of the operator can vary according to the location, the luminance, the contrast, or some other attribute of the image data at a point or its neighborhood. For example, one can consider that the SE is not fixed but that it locally adapts to the data. However, TI is not necessarily lost in all cases of adaptive morphology; e.g., if we chose our actions based on local structure or content, TI is kept *iff* the way in which we select a local SE is itself TI.

Next we outline the main ideas in some major theoretical contributions on how to go from translation-invariant morphology to adaptive morphology. In this paper we mainly focus on flat MM.

2.1. Structuring Element Map (SEM)

A general framework for adaptive morphology, in the Euclidean space, is the concept of the *structuring element map (SEM)*, also known as 'structuring function', proposed in [26, ch.2,9]. In this case, we have not a fixed but a spatially-varying SE, i.e., a map $\mathcal{A}$ that assigns a possibly different set or function $\mathcal{A}(x)$ at each point x of space $\mathbb{E}$. This allows for the following spatial adaptivity rules: (1) *Adaptive Window*, where the operators are flat and use a *Spatially-Varying (SV)* set-valued SEM $\mathcal{A} : \mathbb{E} \to \mathcal{P}(\mathbb{E})$; e.g., the SV flat dilation and erosion [3, 4]

$$\begin{array}{rcl} \mathcal{D}_{\mathcal{A}}(f)(x) & = & (f \oplus \mathcal{A})(x) = \bigvee_{y \in \mathcal{A}(x)} f(x-y) \\ \mathcal{E}_{\mathcal{A}}(f)(x) & = & (f \ominus \mathcal{A})(x) = \bigwedge_{y \in \mathcal{A}(x)} f(x+y) \end{array} \tag{3}$$

(2) *Adaptive Kernel*, where the operators (1) use an SV gray kernel g. (3) *Adaptive Weighted* operators whose SEs are functions g with a fixed support B but SV weights/values.

Since the introduction of the concept of SEM, the interest of the scientific community for adaptive morphology has continuously increased.

2.2. Group-invariant MM

Consider a group $\mathbb{T}$ of automorphisms on the signal domain $\mathbb{E}$. An operator ψ, acting on a complete lattice $\mathcal{L}$ of signals with domain $\mathbb{E}$, is called $\mathbb{T}$-invariant if $\psi\tau = \tau\psi$ for all $\tau \in \mathbb{T}$. The prototypical case is $\mathbb{T}$ to be the Euclidean translation group; then $\mathbb{T}$-invariant means a TI operator in the classic sense. However, as stated in [23], "for certain applications the use of translation-invariant transformations is not appropriate in view of an internal structure which does not possess translation symmetry". Thus, we can restrict to morphological operators that are invariant under a different group of transformations, and talk about *group-invariant morphology* or *group morphology* [13,22]. The theory was developed for polar morphology in [23], for general commutative symmetry groups in [12, 13], and for general non-commutative symmetry groups in [22] including the Euclidean motions (rotations and translations), and perspective transformations for 3D-to-2D projections.

Such group-invariant signal dilations (resp. erosions) are generalized supremal (resp. infimal) convolutions that are adaptive since they are equivalent to using an SEM, as explained next.

2.3. Kernel/Basis Representation Theory

Every increasing TI set operator Ψ can be represented as a union of erosions by its kernel elements and as an intersection of dilations, according to Matheron's theorem. This theory was extended to TI function operators in [15], where also a basis representation was introduced for set and function operators. The above theories of TI operators were extended in [3, 4] for spatially-varying morphology by using representations with supremum and infimum of SV erosions and dilations respectively. For example, let a flat operator ψ (with corresponding set operator Ψ) have a kernel $\mathrm{Ker}(\Psi) := \{\mathcal{A} : x \in \Psi(\mathcal{A}(x))\ \forall x \in \mathbb{E}\}$. Then it is increasing iff it can be represented as supremum of SV erosions by SEMs in the kernel:

$$\psi(f) = \bigvee_{\mathcal{A} \in \mathrm{Ker}(\Psi)} \mathcal{E}_{\mathcal{A}}(f) \tag{4}$$

Thus, any increasing operator can be decomposed into a sup of adaptive erosions (or inf of adaptive dilations). These results unify the adaptive morphological operators based on SV neighborhoods with those based on group morphology. For example, polar morphology [23] and affine morphology [16] were shown in [3] to correspond to SV morphological operations with specific choices for an SEM.

2.4. Level Adaptive MM

A flat operator (2) uses a fixed set operator for each intensity level v; it is clearly a VTI operator. We can avoid this vertical invariance as follows. Given a family $\{\psi_v\}$ of increasing set operators, we can build a *level-adaptive* operator, called *semi-flat* operator in [12],

$$\Phi(f) = \bigvee_{v \in \mathbb{R}} v \cdot \psi_v(\chi_v(f)) \tag{5}$$

that uses a different set processing operator ψ_v for each level, provided that $\{\psi_v\}$ is a decreasing family of increasing set operators.

3. SPECIFIC APPROACHES FOR ADAPTIVE MM

Several ways have been explored to decide how to use local characteristics of the image (including geometrical, statistical or radiometric information) in order to locally design the SE (in shape and/or size) at each point of the space. Most works deal with improving the visual image quality by designing filters that privilege intra-region smoothing rather than inter-region smoothing. This idea motivated the introduction of nonlinear filters in [20] based on anisotropic diffusion. In the framework of MM, one can also consider anisotropic neighborhoods. This can be achieved via several approaches, outlined below. Most of them can be explained using the SEM concept. The last one is based on level adaptivity.

Distance-based: Neighborhoods are defined as set of points at a distance from the center lower than a threshold, the distance being chosen for its capacity to detect image edges. At least two approaches were studied in this direction: the use of a weighted graylevel distance led to the concept of *morphological amoebas* [14]. In this approach the distances are computed on a *pilot image*, which is a smoothed version of the original image. Another approach [10] uses a similarity measure between the image pixels that combines both spatial and tonal information. The similarity is defined as a decreasing function of the *geodesic time*, which is computed by integrating the image gradient magnitude along paths. As in [14], the geodesic time is a weighted distance transform.

Connectivity-based: Adaptivity is omnipresent in connected morphological operators [19, 25, 27], which emphasize connectivity in images instead of geodesic distances. For instance, area openings [6] at scale λ are geometry-adaptive filters: the size of the structuring element is linked to the area λ of the connected components of image. The volume opening [28] at scale λ was explained in [17] as a level-adaptive connected filter (5), for which ψ_v equals at each intensity level v an area opening with parameter λ/v.

Adaptive Neighborhoods: In [5, 9], given some criterion mapping h (expressing local radiometric, morphological, or geometrical information) and a tolerance $m > 0$, at each point $x \in \mathbb{E}$ an adaptive neighborhood $V_m^h(x)$ is defined that contains all points y with $|h(y) - h(x)| \leq m$ and is connected. Obviously, its shape and size vary spatially and adapt to the local image characteristics around the seed point. Then, one can build an SEM that provides an auto-reflected collection of adaptive SEs [3, 5, 9]

$$\mathcal{A}(x) = \bigcup_{z \in \mathbb{E}} \{V_m^h(z) : x \in V_m^h(z)\} \tag{6}$$

and use this to construct SV dilations and erosions as in (3).

Adaptive Rank operators: These correspond to rank filters whose operational window is a graylevel SEM, i.e. an adaptive set of weighted signal values to be ranked. A class of adaptation rules developed in [24] is based on minimizing a local MSE or MAE error via steepest descent using LMS-like algorithms. These approaches weight the SE using a rank-sum arithmetic. However, in rank filters it is also possible to use weights as repetition numbers of the signal values. The adaptation of such filters was approached in [21] by using the procedure of [24].

Viscous MM: Edge preservation is not always a goal in image processing. In many situations, e.g. in contour detection, new edges will be created in the image in order to close or regularize the existing contour lines. The problem is then to modify the local geometry of the image at places where regularization is required while preserving the precision of the data at places where they are accurate. This motivated [18, 31] to introduce level-adaptive morphological filters, associated with an adaptive SE of fixed shape and whose size adapts w.r.t. the local image intensity or contrast. These so-called *viscous operators* are no longer VTI. The viscous dilations β and erosions α proposed in [32] process different level sets by different scales [δ_r and ε_r denote flat dilation and erosion by a disk of radius r]:

$$\beta(f) = \bigvee_v v \cdot \delta_{M-v}[\chi_v(f)], \quad \alpha(f) = \bigwedge_v v \cdot \varepsilon_{M-v}[\chi_v(f)] \tag{7}$$

where $M = \sup_x f(x)$. Some viscous operators are semi-flat.

4. GEOMETRIC PDES AND ADAPTIVITY

Modeling multiscale image filtering via PDEs offers continuous scale evolution, better and more intuitive mathematical modeling, connections with physics, and closer approximation to the continuous geometry of the problem. The most famous is the linear isotropic diffusion PDE $\partial_t u = \nabla^2 u$, which corresponds to Gaussian convolutions. To avoid the edge blurring of Gaussian scale-space, a nonlinear anisotropic diffusion PDE was proposed in [20]

$$\partial_t u = \mathrm{div}(g(\|\nabla u\|)\nabla u) = g\nabla^2 u + \nabla g \cdot \nabla u \tag{8}$$

where g is a smooth nonincreasing function that inhibits smoothing at strong edges (acting like a varying diffusion coefficient), whose purpose is to favor intra-region over inter-region smoothing. One could approximately interpret (8) as a linear convolution by a spatially-varying kernel. Tensor generalizations of (8) in [30, 34] indeed admit such an approximate interpretation as adaptive convolutions by anisotropic Gaussians whose major axes are parallel to the eigendirections of the local image structure tensor. A conceptually similar adaptation based on the local Hessian was used in some morphological filters [29]. Nonlocal means [2] is an adaptive neighborhood filter that smooths by averaging pixels not by spatial but by graylevel proximity; it is asymptotically equivalent to (8).

If we attempt to also model level-adaptive MM with differential rules, the main objective is to find PDEs for the viscous dilation and erosion (7), since they are the building blocks of viscous operators. In [17] we introduced the following scale-space PDE models

$$\begin{aligned} \partial_t u(x,t) &= (M - u(x,t))^+ \|\nabla u\| \\ \partial_t w(x,t) &= -(M - w(x,t))^+ \|\nabla w\| \end{aligned} \tag{9}$$

with the original image as initial condition: $u(x, 0) = w(x, 0) = f(x)$. The PDE for u or w generates the viscous dilation $\beta(f)$ or erosion $\alpha(f)$ respectively. Indeed, the level curves of the function $\phi(x, t)$ that satisfies the PDE $\partial_t \phi = c(x, t)\|\nabla \phi\|$ move on the plane with normal speed $c(x, t)$. Thus, the isoheight curve of $u(x, t)$ at level v moves with speed $M - v$.

5. COMPUTATIONAL METHODS

A large part of the success of mathematical morphology in the engineering community is due to the algorithmic developments. Very efficient algorithms have been proposed for TI morphological operators for both binary and graylevel images, and for both software and hardware implementations. However, "supporting a variable structuring element shape imposes an overwhelming computational complexity, dramatically increasing with the size of the structuring element", as mentioned in [11]. Thus, algorithms addressing the case of spatially adaptive SEs are still very limited.

In [8] locally adaptable binary erosions and dilations were implemented as a variant of distance transformation algorithms. However, the only possible extension of the strategy to the graylevel case is based on a decomposition of the function into its thresholds. This considerably decreases the strategy efficiency.

For implementing binary adaptive morphology, an algorithm in [11] was limited to non-centered adaptive rectangles with low memory requirement and latency. In the graylevel case, a fast algorithm was proposed in [7] for spatially-varying SEs with adaptable shape and size based on a decomposition of the SEs in smaller 2D subelements. Tree-based fast algorithms for connected filters can be found in [19, 25].

Finally, the PDEs (9) for level-adaptive MM were implemented in [17] using fast nonlinear difference equations.

6. APPLICATIONS AND DISCUSSION

Noise reduction: So far, the main application domain of adaptive filtering in both linear and nonlinear cases is certainly the noise reduction. Denoising is to remove noise as much as possible while preserving useful information as much as possible. Adaptivity can be helpful in denoising applications since it allows to design the filter in accordance with the model of noise while adapting the filtering parameter to the image content in such a way to preserve features of main importance in the image, for instance edges, contrast, local geometry, color appearance.

Perspectivities and other geometric deformations: One of the earliest applications that requested the use of SEs of variable size is the analysis of images from traffic control cameras [1]. Because of the perspective effect, vehicles at the bottom of the image are closer and appear larger than those higher in the image. Hence, the SE should follow a law of perspective, for example, vary linearly with its vertical position in the image. This example is nicely supplemented by many other examples where more complex variations of the relevant SE size are required. For instance, in [23] a photograph of the trees in a forest was taken by putting the camera at ground level and aiming towards the sky. This case requires a polar structure where the size of the structuring element increases with the distance from the center of the image. In [33], the interest was in range imagery, a modality where the value of each pixel is the distance to the imaging device. Thus, the perspective requires to consider structuring elements that locally adapt their size to the image content.

Viscous filters and viscous watershed: In many segmentation applications, variational methods are the best because they make the balance between two requirements: a necessity of precision at place where contours are sure and a certain amount of modeling at place where the contour information is missing. In [18, 31] similar segmentations were obtained via connective segmentation paradigms, as the watershed transform, by computing it on images with regularized contours produced by intensity- or contrast-adaptive filters called *viscous filters*. By viscous filtering, contours of high luminance or of high contrast are left unchanged (which ensures a good precision of the final segmentation) while points of low luminance are hardly closed (which ensures the regularization of the final segmentation).

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