

PDE-BASED MODELING OF IMAGE SEGMENTATION USING VOLUMIC FLOODING

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ABSTRACT

The classical case of morphological segmentation is based on the watershed transform, constructed by flooding the gradient image, which is seen as a topographic surface, with constant height speed. Changing the flooding criteria, (e.g constant-speed height, area or volume) yields different segmentation results. In the field of PDEs and curve evolution the classic watershed transform can be modelled as the solution of an eikonal PDE. In this paper we model the watershed segmentation based on a volume flooding criterion via a different eikonal PDE. Then we solve this PDE using the fast marching method, which is a specific algorithm from the methodology of level sets. In addition, we attempt to exploit the advantages of image segmentation using PDE-based volume flooding over the classic height flooding.

1. INTRODUCTION

Segmentation is one of the most difficult tasks in image processing, as it is application dependent and requires to some extent a semantic understanding of the image. The morphological segmentation methodology, based on the watershed transform and markers, has been successfully used, both for interactive as for automated segmentation ([1], [13]). The segmentation process starts with creating flooding waves that emanate from the set of markers and flood the image gradient surface. The points where the emanating waves meet each other form the segmentation boundaries. The simplest markers are the regional minima of the gradient image. Very often, the minima are extremely numerous, leading to an oversegmentation. For this reason, in many practical cases, the watershed will take as sources of the flooding a smaller set of markers, which have been identified by a preliminary analysis step as inside particles of the regions or objects that need to be extracted via segmentation. The advantage of the aforementioned method is robustness: the result is independent of the shape or the placement of the markers in the zones of interest. The result is obtained by a global minimization implying both the topography of the surface and the complete set of markers.

Based on the criteria governing the flooding process, different types of segmentation can occur with varying characteristics in their results. By the term ‘flooding criterion’ we refer to a characteristic that all lakes (associated with the flooding sources) share with respect to water, such as altitude/height (contrast criteria), area (size criteria) or volume (contrast and area criteria) [6]. The most common type of flooding that forms the base of all traditional morphological segmentation schemes is the one obtained when the

This work was supported by the Greek General Secretariat for Research and Technology and by the European Union under the program IIENEΔ – 2001.

water altitude variation is at the same level for all lakes, known as uniform height flooding.

In mathematical morphology uniform flooding segmentation has been implemented via immersion simulations [13] and hierarchical queues [1]. Watershed segmentation has also been modelled via the eikonal PDE ([8], [5], [2], [7]). Modelling the watershed via the eikonal has the advantage of a more isotropic flooding but it also introduces some challenges in the implementation. In general, efficient algorithms to solve time-dependent eikonal PDEs are the narrow-band level sets methods [9, 11]. An even more efficient algorithm for stationary formulations of eikonal PDEs is the fast marching method (FMM) introduced in Sethian ([11], [12]). In the field of curve evolution and PDEs, watershed segmentation has also been modelled and implemented via the eikonal PDE and level sets by Maragos & Butt [3], [4].

In this paper we explore a different flooding criterion for the watershed segmentation, in particular flooding with uniform volume speed as introduced by Meyer [6], but with a new PDE model and a corresponding numerical approximation. Our first contribution is to model the volume flooding process via a time-dependent eikonal-type PDE and level sets. Further, for solving this PDE we propose an efficient implementation based on a variation of the fast marching method. Finally, we provide experimental results to illustrate the new segmentation method and compare it with the classic watershed segmentation.

2. TYPES OF WATERSHED FLOODINGS AND PDES

As it was mentioned above, different types of segmentation can be obtained by varying the flooding criterion. In our research work, we are concerned about two cases: (1) Uniform Height Watershed Flooding and (2) Uniform Volume Watershed Flooding.

2.1. Uniform Height Flooding

Let's consider the case of flooding the 1D function f shown in fig.1(a). First we pierce this function at one of its regional minima, as shown in fig.1(a), and then we immerse it in water with constant vertical speed. In this case the water altitude at each time instance is uniform. By ΔH we refer to the height difference from one time instance to another. When the flooding of the function f is done with uniform height speed we have $\frac{\Delta H}{\Delta t} = \text{const} = c$. Given that $\tan(\theta) = \frac{\Delta H}{\Delta L} = |f'|$ we have:

$$\frac{\Delta H}{\Delta t} = |f'| \frac{\Delta L}{\Delta t} \implies V = \frac{\Delta L}{\Delta t} = \frac{c}{|f'|} \quad (1)$$

By the term V we refer to the horizontal velocity by which the level sets of the function f propagate in time.

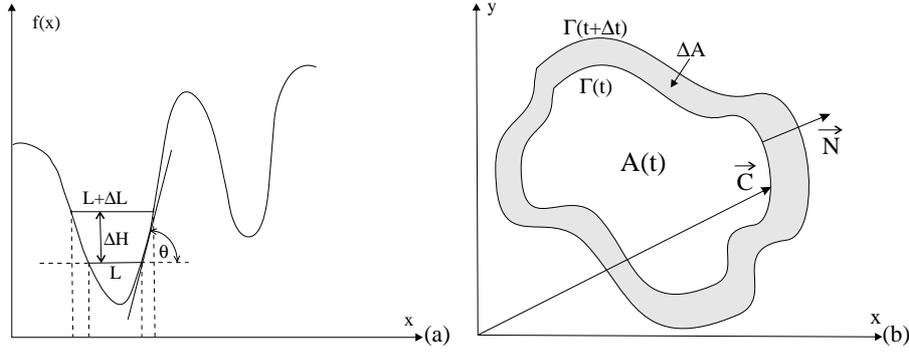


Fig. 1. (a) Lakes of 1D function; $L(t)$ is length of level sets. **(b)** Planar Projection of a lake of 2D function; $A(t)$ is area of level sets.

Let's now consider the case a 2D function. Shown in fig.1(b) is the planar projection of a lake of a 2D function, flooded under the constraint of uniform height. The boundary of the lake at a time instance t is the set $\Gamma(t)$ of points of the closed planar curve represented by its position vector $\vec{C}(t)$. This curve models the propagation of a wave emanating from the lake. Based on Eq.(1) and considering that in 2D shapes (a) ΔL becomes the displacement normal to the level curves of the function and (b) $|f'|$ becomes $\|\nabla f\|$, we model the propagation of the curve \vec{C} with the following PDE:

$$\frac{\partial \vec{C}}{\partial t} = \frac{c}{\|\nabla f\|} \cdot \vec{N} \quad (2)$$

where \vec{N} is the normal vector to the curve.

Now, following the level set formulation proposed by Osher & Sethian [9], let us embed this evolving planar curve as the zero-level curve of an evolving space-time function $\phi(x, y, t)$; i.e., $\Gamma(t) = \{(x, y) : \phi(x, y, t) = 0\}$. Then the PDE that governs the evolution of the obtained level function is:

$$\frac{\partial \phi}{\partial t} = V(x, y) \|\nabla \phi\| \quad (3)$$

where $V(x, y)$ is the space-dependent speed function given by $V(x, y) = c/\|\nabla f(x, y)\|$.

2.2. Uniform Volume Flooding

Let's now investigate the case where the flooding is done with uniform volume speed inside all lakes. The main difference here compared to traditional watershed segmentation explained in sec.2.1 is that during the flooding process the water height is not at same level for all lakes. However, what remains the same at each stage is the volume change rate of water in each lake. This means that the variation of water volume in the lakes is constant, retaining the balance between area and contrast. Following the same procedure as above we flood the 1D function f of fig.1(a). We then have that $L \frac{\Delta H}{\Delta t} = \text{const} = c$, which yields the horizontal velocity V :

$$\frac{\Delta L}{\Delta t} = \frac{\Delta H}{\Delta t} \frac{1}{|f'|} \implies V = \frac{\Delta L}{\Delta t} = \frac{c}{L(t) |f'|} \quad (4)$$

We then consider the 2D case of fig.1(b) where the curve \vec{C} models the wave emanating from a lake flooded under the constraint of uniform volume speed. In this case $L(t)$ becomes $A(t)$,

which is the area enclosed by the propagating wave at time t , and we obtain the following PDE that governs the evolution:

$$\frac{\partial \vec{C}}{\partial t} = \frac{c}{A(t) \|\nabla f\|} \cdot \vec{N} \quad (5)$$

The equivalent PDE level function is:

$$\frac{\partial \phi}{\partial t} = V(x, y, t) \|\nabla \phi\| \quad (6)$$

where $V(x, y, t) = c/(A(t) \|\nabla f(x, y)\|)$ is the time and space-dependent speed function. The product $A(t) \|\nabla f\|$ is a measure of volume. As a consequence, the speed of the evolving curve is inversely proportional to the volume of the sources of the flood, so the flood is slowed down by the factor $1/V$. This means that during the flooding process lakes with large volume are filled up by water slowly whereas lakes with small volume are filled up quickly. The evolution PDE (5) can be viewed as a dilation with a varying radius structuring element. The radius $1/(A(t) \|\nabla f(x, y)\|)$ of the structuring element depends on space (x, y) and time t .

3. FLOODING IMPLEMENTATION USING FAST MARCHING METHODS

The PDEs derived in sec.(2.1), (2.2) are time-dependent PDEs. Since in both cases the speed of the evolving front is one-directional, that is it has constant sign, we can consider the stationary formulation of the embedding level function evolution PDEs (3),(6), with positive speed V , which is known as 'eikonal' PDE. Namely, if

$$T(x, y) = \inf\{t : \phi(x, y, t) = 0\}$$

is the *minimum time of arrival* at which the zero-level curve of the level function $\phi(x, y, t)$ described by Eq.(3),(6) crosses (x, y) , then

$$\|\nabla T(x, y)\| = \frac{1}{V}$$

Considering the two different cases of flooding discussed earlier, the resulting 'stationary' eikonal-type PDEs are:

$$\begin{aligned} \text{Height Flooding:} & \quad \|\nabla T(x, y)\| = \|\nabla f\|/c \\ \text{Volume Flooding:} & \quad \|\nabla T(x, y)\| = A(T) \|\nabla f\|/c \end{aligned} \quad (7)$$

A methodology used to solve such PDEs was proposed in [12] and it belongs to a wider category of efficient algorithms called

'Narrow Band Level Set Methods' [11]. The specific methodology, known as 'Fast Marching Method' (FMM), aims at approximating the solution of stationary eikonal PDEs in order to reduce the computational complexity of solving time-dependent eikonal PDE problems via curve evolution. The main advantages of this methodology are that it automatically selects solutions that include non-differentiability in natural ways, and it is extremely efficient computationally. The central idea is to track only a narrow band of pixels at the boundary of the evolving wavefront. For this purpose a narrow band is defined, including all pixels one grid point away. The pixel (x, y) in the narrow band that produces the minimum $T(x, y)$ is propagated first. The minimum $T(x, y)$ is estimated by solving a quadratic equation. The main algorithm is fully described in [12].

The propagation of all waves (associated with the relevant sources) begins simultaneously. At places where two or more fronts meet, a dam is erected to specify the segmentation line. Whereas in the case of uniform height flooding FMM is easy to implement as done in [4], in the case of uniform volume flooding solving the PDE of Eq.(7) has the peculiarity of the pseudo time varying term $A(T)$. This is why the fast marching algorithm has to be implemented so it takes under consideration the time-dependent area variations of the fronts. Every time we 'march' a pixel forward the area of the relevant front is increased. Consequently, the corresponding area term of the front has to be updated.

4. RESULTS AND DISCUSSION

Different experiments were undertaken in order to investigate the advantages of using volume flooding, and find out the differences it has over the classic one. Some of the results are presented in fig.2, fig.3 and fig.4. The images used to test the proposed methodology exploit the need for using this type of flooding. In fig.2(a) a synthetic image is presented. It was created by taking the distance transform of a binary image of four randomly shaped objects and adding an arbitrary constant to each one, thus producing a grey level image. Four different cases are examined. At first, uniform volume flooding is applied directly to the original image and then to its gradient. Secondary, uniform height flooding is applied to the initial image and its gradient. The segmentation results, presented in fig.2(d)-(e), exploit the basic property of volume flooding, that is, it retains the balance between area and contrast. Using four markers (one for each object) that have been set manually, we flood the image from these predefined sources. When uniform volume flooding is applied, we see that the object that is eventually lost is the one with the smallest volume (area and contrast). On the other hand, when uniform height is applied, the object that is finally lost is the one with the lowest contrast, which happens to be the one with the biggest area. Additionally we can say that uniform volume flooding behaves better when it is applied directly to the image of interest and not its gradient. Judging by this experimental result we can say that volume flooding leads to more balanced results.

In fig.3 and fig.4 are presented the results of the aforementioned types of flooding applied to real images. We have used the image of a parrot as well as a medical image which is a thorax MRI. Differences in the images' intensity create lakes with different volume (because their area and altitude - intensity are not the same). In fig.3(a)-(d), fig.4(a)-(d) are illustrated the results of the preliminary stages before the actual segmentation. These are: image simplification in order to create more flat lakes and feature

extraction, such as edge gradient and marker set. Image simplification is achieved by applying non-linear filtering [10] (ASF based on reconstruction) and feature extraction is accomplished using morphological techniques [10]. The marker set is a binary set defined manually where each particle marks a region of interest in the image. The segmentation results are presented in fig.3(e)-(h), fig.4(e)-(h) respectively. As it can be seen, volume flooding generally behaves better. It seems to be more balanced since it takes under consideration not only the contrast criteria but area criteria as well. It should be mentioned that whereas uniform height flooding should be applied to the gradient of the image of interest, volume flooding produces better results when applied directly to the image not to its gradient. In fig.3(i)-(l) we see the results of height and volume segmentation applied directly on the initial image, without any simplification preprocessing. Volume flooding behaves better than height flooding, especially when applied on the image function and not on its gradient.

In conclusion, we can say that volume flooding exploits both the contrast and the size properties of the objects present in an image. It can be considered as a useful segmentation tool in cases where we want to keep the balance between the aforementioned image properties and it can give good results in cases that contrast-based driven segmentation fails. In addition, the PDE implementation has the advantage of a more isotropic flooding. There is the consideration of adding an attraction term to the PDE describing the evolution of the curve in Eq.(5) in order to make a more robust, efficient and active contour-like segmentation methodology.

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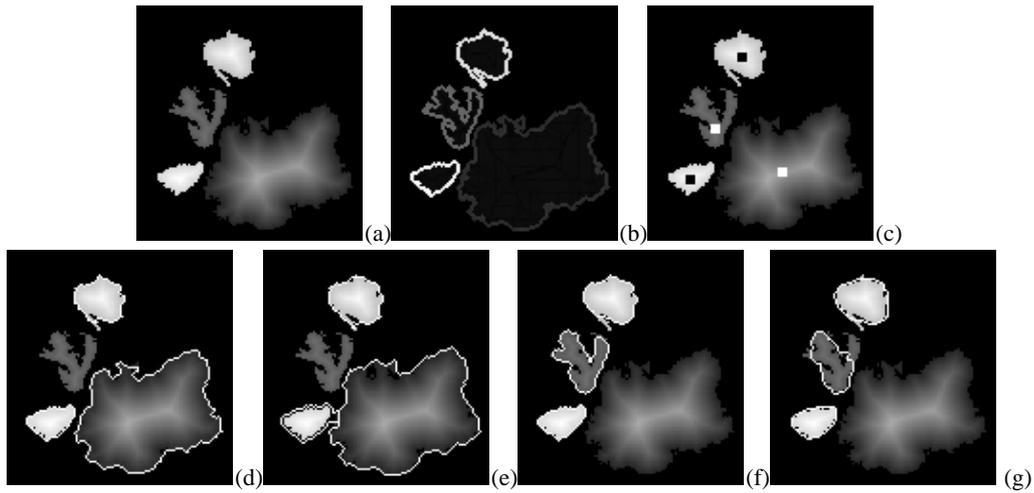


Fig. 2. (a) Synthetic image f (b) Gradient $\|\nabla f\|$ (c) Markers (d) Vol. Flooding of f (e) Vol. Flooding $\|\nabla f\|$ (f) Height Flooding of f (g) Height Flooding of $\|\nabla f\|$.

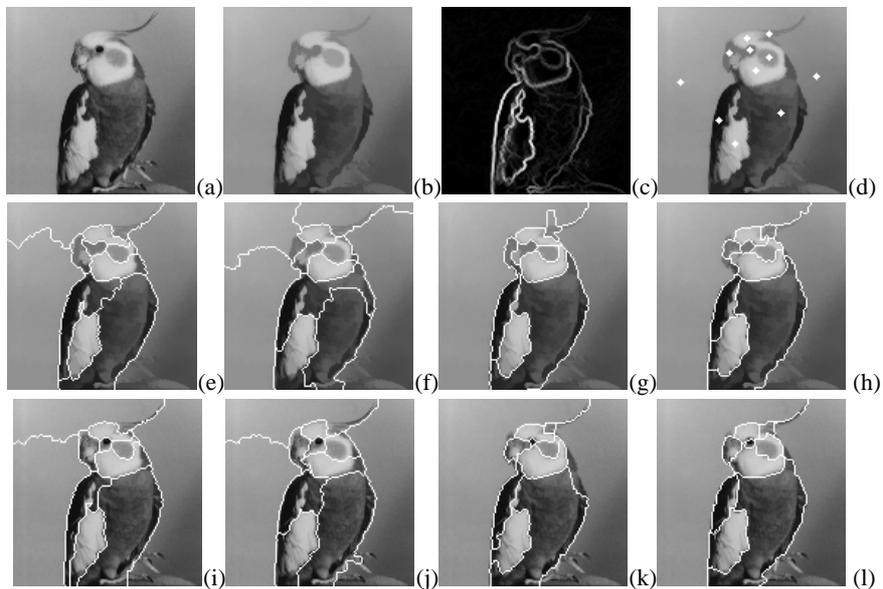


Fig. 3. (a) Parrot image f (b) Simplified image g (c) Gradient $\|\nabla g\|$ (d) Manual markers (e) Vol. Flooding of g (f) Vol. Flooding of $\|\nabla g\|$ (g) Height Flooding of g (h) Height Flooding $\|\nabla g\|$ (i) Vol. Flooding of f (j) Vol. Flooding of $\|\nabla f\|$ (k) Height Flooding of f (l) Height Flooding $\|\nabla f\|$.

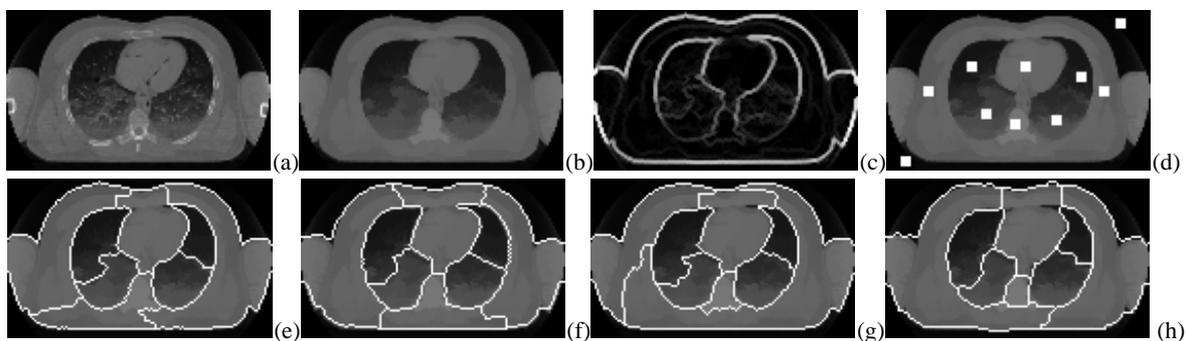


Fig. 4. (a) MRI thorax f (b) Simplified image g (c) Gradient $\|\nabla g\|$ (d) Manual markers (e) Vol. Flooding of g (f) Vol. Flooding of $\|\nabla g\|$ (g) Height Flooding of g (h) Height Flooding of $\|\nabla g\|$.