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Signal Processing 69 (1998) 81–91

SIGNAL PROCESSING

Harmonic analysis and restoration of separation methods for periodic signal mixtures: Algebraic separation versus comb filtering[1]

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Received 3 November 1997; received in revised form 18 May 1998

Abstract

The problem of separating a mixture of periodic signals into its constituent components occurs in sound detection, biomedical signal processing, and in communications. Existing approaches to solving it are either based on harmonic selection in the frequency domain or on linear comb filtering in the time-domain. In this paper, the recently proposed matrix algebraic separation approach is analyzed in the frequency domain. The insight obtained via this analysis leads to the development of harmonic restoration techniques that fill in the information missing at the harmonics shared by the components and also to the development of constraints on the carrier frequencies and bandwidths for narrowband, bandpass, and periodic AM–FM components for minimum information loss. The restored methods are then applied to mixtures of sines and AM–FM signals. Differences between this improved approach and a similar improvement of the comb filtering approach are also emphasized. © 1998 Elsevier Science B.V. All rights reserved.

Zusammenfassung

Das Problem der Zerlegung einer Überlagerung periodischer Signale in seine Komponenten taucht in akustischer Detektion, biomedizinischer Signalverarbeitung und Kommunikation auf. Die vorhandenen Lösungen beruhen auf harmonischer Selektion im Frequenzbereich oder linearer Kammfilterung im Zeitbereich. In der vorliegenden Arbeit wird eine Analyse im Frequenzbereich vorgestellt, die die kürzlich vorgeschlagene Methode der matrixalgebraischen Separation untersucht. Die dadurch gewonnenen Ergebnisse führen sowohl zur Entwicklung von harmonischen Restaurationstechniken, die verlorene Information in den gemeinsamen Harmonischen der Komponenten ersetzen, als auch zur Entwicklung von Beschränkungskriterien an die Trägerfrequenzen und Bandbreiten von Schmalband-, Bandpaß- und periodischen AM–FM Komponenten, mit dem Ziel, minimalen Informationsverlust zu erhalten. Die Restaurationsmethode wird auf eine Mischung von Sinus- und AM–FM Signalen angewandt. Darüberhinaus werden die Unterschiede zwischen dieser verbesserten Methode und einer ähnlichen Verbesserung des Kammfilter-Ansatzes hervorgehoben. © 1998 Elsevier Science B.V. All rights reserved.

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[1] This work was supported by the U.S. National Science Foundation under Grants MIP-93963091 and MIP-9421677.

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PII: S0165-1684(98)00088-7

Résumé

Le problème de la séparation d'un mélange de signaux périodiques en ses constituants se retrouve en détection de son, en traitement des signaux biomédicaux, et en communications. Les approches existantes de résolution de ce problème sont basées soit sur une sélection harmonique dans le domaine fréquentiel soit sur un filtrage linéaire "en peigne" dans le domaine temporel. Dans cet article, l'approche récemment proposée de séparation algébrique matricielle est analysée dans le domaine fréquentiel. L'éclairage obtenu à l'aide de cette analyse conduit au développement de techniques de restauration harmoniques qui pallient le manque d'information sur les harmoniques partagées par les composantes et également au développement de contraintes sur les porteuses en fréquence et les bandes passantes des composantes bande-étroite, passebande, et AM–FM périodiques pour une perte d'information minimale. Les méthodes de restauration sont ensuite appliquées à des mélanges de sinusoïdes et de signaux AM–FM. Les différences entre cette approche améliorée et une amélioration similaire de l'approche par filtrage en peigne sont également soulignées. © 1998 Elsevier Science B.V. All rights reserved.

Keywords: Periodic mixture separation; Algebraic separation; Comb filtering; Harmonic reassignment

1. Introduction

Separation of an additive mixture of periodic or quasi-periodic signals into its constituent components, where one or both components contain useful information (hereafter referred to as the separation of periodic mixtures (SPM) problem) is an important signal processing and detection task encountered: (i) when dealing with the recovery of multiple sinusoids in noise [5], (ii) in biomedical signal processing problems such as separating a fetal ECG signal from a composite ECG signal which also contains the maternal ECG signal [6], (iii) when dealing with interference rejection in communication systems that transmit information using narrowband bandpass signals [2], and (iv) in the area of concurrent vowel separation and in the process of separating a speech signal from an interfering speech signal [1,3].

The SPM problem can be approached as a frequency domain harmonic reassignment problem, i.e., selection of the harmonics that belong to either component from the composite signal spectrum. Time-domain estimates are obtained via the inverse FFT of their respective spectral impulse trains [3]. The SPM problem can also be approached as a time-domain filtering problem where component separation is accomplished via comb filtering [1]. Matrix algebraic separation of periodic signal mixtures was introduced in [7,8], where the components of an additive mixture of two periodic signals were separated using their periodicity and samples of the composite signal in a matrix framework via linear algebra techniques. As shown in [4], this matrix algebraic separation (MAS) technique can separate a mixture of two periodic narrowband bandpass components even when there is complete spectral overlap. Although a time-domain analysis of the MAS algorithm was presented in [4], there is lack of understanding of this technique in the frequency domain.

In this paper, we present a harmonic analysis of the SPM problem and then analyze the automated approaches of the MAS algorithm [7,8] and linear comb filtering [1] in this context both in the time and frequency domains. Using the insight gained from this analysis we then propose harmonic restoration techniques for both of the separation methods to improve their performance. These improved methods are then applied to mixtures of sinusoidal and periodic AM–FM signals.

2. Harmonic analysis of the general separation problem

2.1. Common harmonics

The fundamental step in both the comb filtering and the MAS algorithms is the modeling of the components as periodic signals with fundamental period N_i:

$$x_i[n] = x_i[n + N_i] \Leftrightarrow x_i[n] - x_i[n - N_i] = 0. \quad (1)$$

Periodicity of the components implies a discrete spectrum with impulses at multiples of its fundamental frequency $2\pi/N_i$:

$$X_i(\Omega) = 2\pi \sum_{m=-\infty}^{\infty} a_{im}\delta\left(\Omega - \frac{2\pi m}{N_i}\right), \quad i = 1,2. \tag{2}$$

On the other hand, the composite signal $x[n] = x_1[n] + x_2[n]$ is also periodic with a repetition period $P = \mathrm{lcm}(N_1,N_2)$ and has a discrete spectrum

$$X(\Omega) = X_1(\Omega) + X_2(\Omega) = 2\pi \sum_{m=-\infty}^{\infty} b_m\delta\left(\Omega - \frac{2\pi m}{P}\right). \tag{3}$$

The inverse problem of component separation becomes equivalent to the task of obtaining the sequences $\{a_{im}\}$ from the sequence $\{b_m\}$, i.e., reassigning the spectral content of the composite signal to the components. If $R = \gcd(N_1,N_2) = 1$, then the harmonics of the fundamental frequencies of the two components are distinct[2] and reassignment is straight-forward. However, when $R > 1$, the component spectra share harmonics and confusion arises regarding the reassignment of the R common harmonics at the frequencies

$$\Omega_k = \left(\frac{2\pi}{R}\right)k, \quad k = 0,1,\dots,(R-1). \tag{4}$$

At these frequencies the spectral impulse amplitudes of the components are superimposed:

$$A_{1k} + A_{2k} = B_k, \quad k = 0,1,\dots,(R-1), \tag{5}$$

where $\{A_{ik}\}$ and $\{B_k\}$ are the spectral impulse amplitude sequences of the ith component and the sum at $\Omega = \Omega_k$. The superposition of the spectral impulse amplitudes at $\Omega = \Omega_k$ results in a loss of information at these frequencies. All separation algorithms more or less can recover the distinct harmonics from the composite spectrum, however, they are confronted with the difficult task of recovering the individual impulse amplitudes $\{A_{ik}\}$ from their sum $\{B_k\}$.

[2] Note that the dc frequency, $\Omega = 0$, is always a shared harmonic for any R.

2.2. Four basic options and the ratio method

The basic options for reallocation of the spectral impulse amplitudes at $\Omega = \Omega_k$ are classified as

OP1: Give it entirely to the second component: $A_{2k} = B_k$, $A_{1k} = 0$, $\forall k$.
OP2: Give it entirely to the first component: $A_{1k} = B_k$, $A_{2k} = 0$, $\forall k$.
OP3: Give them to both components in proportion to a ratio $\lambda_k = A_{1k}/A_{2k}$, $\lambda_k \in \mathbb{C}$.
OP4: Give them to neither component: $A_{1k} = A_{2k} = 0 \Leftrightarrow B_k = 0$, $\forall k$.

Note that OP4 is generally a bad option because it may result in a violation of the known information in Eq. (5) when $B_k \neq 0$. If we allow the parameter λ_k in OP3 to assume infinite values, i.e., $\lambda_k \in \mathbb{C} \cup \{\infty\}$ then OP3 includes OP1 and OP2 as special cases when $\lambda_k = 0$ and ∞:

$$\begin{aligned} &\text{OP1: } \lambda_k = 0 \Leftrightarrow A_{1k} = 0, \quad A_{2k} = B_k, \\ &\text{OP2: } \lambda_k = \infty \Leftrightarrow A_{1k} = B_k, \quad A_{2k} = 0. \end{aligned} \tag{6}$$

For the experiments in this paper, we use the same ratio parameter for all $\Omega = \Omega_k$, i.e., $\lambda_k = \lambda$, $\forall k$.

In general, we need additional information to provide a non-arbitrary value of λ. One heuristic approach we have developed to automatically find a satisfactory value of λ is to assume that each component is narrowband and bandpass. One specific class of narrowband bandpass periodic signals that we examined are the class of periodic AM–FM signals whose Carson bandwidth is much smaller than their carrier frequencies. Assuming for simplicity that the information signals are sinusoidal, the composite signal is the superposition of two narrowband bandpass AM–FM/cosine components:

$$\begin{aligned} x[n] &= \sum_{i=1}^{2} A_i[n]\cos\left(\int_0^n \Omega_i[m]\mathrm{d}m + \theta_i\right), \\ \Omega_i[n] &= \Omega_{ci} + \Omega_{mi}\cos(\Omega_{fi}n + \alpha_i), \\ A_i[n] &= A_{ci}[1 + \kappa_i\cos(\Omega_{ai}n + \beta_i)], \end{aligned} \tag{7}$$

where $\Omega_i[n]$ and $A_i[n]$ are the instantaneous frequency and amplitude information signals, Ω_{ci} and A_{ci} are the carrier frequencies and amplitudes, Ω_{fi} and Ω_{ai} are the bandwidths of the frequency and amplitude information signals, and Ω_{mi}/Ω_{ci} and κ_i are the FM and AM relative amounts on a unit scale. If the common harmonics do not lie within the passband of either component, then information loss at these points is not critical[3] and this happens when the components satisfy:

Bandwidth constraint:

$$\overbrace{2(\Omega_{mi} + \Omega_{fi} + \Omega_{ai})}^{\text{Carson Bandwidth}} \leqslant 2\pi/R,$$

Carrier frequency constraint:

$$\min_k |\Omega_{ci} - \Omega_k| \geqslant (\Omega_{mi} + \Omega_{fi} + \Omega_{ai}). \tag{8}$$

The heuristic rule for the reassignment at $\Omega = \Omega_k$ via options OP1 or OP2 in the case of mixtures of two sinusoidally modulated AM–FM signals is to force spectral nulls on the component with carrier frequency furthest from $\Omega = \Omega_k$, i.e.,

$$\begin{aligned} \min_k |\Omega_{c1} - \Omega_k| > \min_k |\Omega_{c2} - \Omega_k| \;\Rightarrow\; \lambda = 0, \\ \min_k |\Omega_{c2} - \Omega_k| > \min_k |\Omega_{c1} - \Omega_k| \;\Rightarrow\; \lambda = \infty. \end{aligned} \tag{9}$$

This heuristic reduces the damage caused by $\Omega = \Omega_k$ falling on or close to either carrier frequency. Another approach to restore some of the lost information at $\Omega = \Omega_k$ is to interpolate the spectral values $\{A_{1k}\}$ of the first component from neighboring spectral values and then to use Eq. (5) to fill in the values $\{A_{2k}\}$ for the second component. This approach is valid when spectral correlation in the neighborhood of $\Omega = \Omega_k$ exists; the inverse FFT is used to obtain time-domain estimates. Only nearest-neighbor interpolation, i.e., linear interpolation of the spectral magnitude from harmonics immediately adjacent to each $\Omega = \Omega_k$ using a 3 point and weights of $\{0.5,0,0.5\}$ is implemented here.

[3] These constraints place the common harmonics in the tail end of the spectrum of the component signal where there is negligible signal energy. The Carson bandwidth of each AM–FM signal component is a conservative estimate of its effective bandwidth.

3. Harmonic restoration of the two separation methods

3.1. Restoration for matrix algebraic separation

Consider a two-component periodic signal $x[n]$, where the fundamental periods of the components are N_1 and N_2 samples, respectively. Relating N samples of the composite signal to the samples of the components yields the *basic separation system*

$$\underbrace{\begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{bmatrix}}_{\boldsymbol{x}} = \underbrace{\begin{bmatrix} \boldsymbol{I}_{N_1} & \boldsymbol{I}_{N_2} \\ \boldsymbol{I}_{N_1} & \boldsymbol{I}_{N_2} \\ \vdots & \vdots \end{bmatrix}}_{\boldsymbol{S}} \underbrace{\begin{bmatrix} x_1[0] \\ \vdots \\ x_1[N_1-1] \\ x_2[0] \\ \vdots \\ x_2[N_2-1] \end{bmatrix}}_{\boldsymbol{z}}, \tag{10}$$

where $\boldsymbol{I}_{N_1}$ denotes the identity matrix of order N_1 and the rank of the separation matrix $\boldsymbol{S}$ is $r(\boldsymbol{S}) = N_1 + N_2 - R$ [7,8]. This information deficiency at $\Omega = \Omega_k$ translates into the rank deficiency of $\boldsymbol{S}$. The extra constraint/equation needed to complete the basic separation system for coprime periods is typically a dc value condition of the form

$$\sum_{n=0}^{N_1-1} x_1[n] = 0. \tag{11}$$

When the periods are not coprime the $R = \gcd(N_1,N_2)$ constraints needed are obtained as

$$\sum_{j=0}^{(N_1/R)-1} x_1[Rj+i] = 0, \quad i = 0,1,\ldots,(R-1). \tag{12}$$

The components are obtained via the least–squares solution to the *augmented separation system*

$$\underbrace{\begin{pmatrix} \boldsymbol{S} \\ \boldsymbol{C} \end{pmatrix}}_{\tilde{\mathbf{S}}} \mathbf{z} = \underbrace{\begin{pmatrix} \boldsymbol{x} \\ \mathbf{0} \end{pmatrix}}_{\tilde{\mathbf{x}}}, \tag{13}$$

where the homogeneous dc value constraints at the scale of R form the constraint matrix $\boldsymbol{C}$. The effective separation system for each of the components is of the form [4]

$$\hat{z} = \tilde{\boldsymbol{S}}^{\dagger}\tilde{\boldsymbol{x}} = (\boldsymbol{S}^{\mathrm{T}}\boldsymbol{S} + \boldsymbol{C}^{\mathrm{T}}\boldsymbol{C})^{-1}\boldsymbol{S}^{\mathrm{T}}\boldsymbol{x},$$
$$\hat{\boldsymbol{x}}_1 = [\boldsymbol{I}_{N_1\times N_1}\boldsymbol{0}_{N_1\times N_2}]\hat{z} = \boldsymbol{S}_1^{\dagger}\boldsymbol{x}, \tag{14}$$
$$\hat{\boldsymbol{x}}_2 = [\boldsymbol{0}_{N_2\times N_1}\,\boldsymbol{I}_{N_2\times N_2}]\hat{z} = \boldsymbol{S}_2^{\dagger}\boldsymbol{x},$$

where the notation $\tilde{\boldsymbol{S}}^{\mathrm{T}}$ stands for the matrix transpose, $\tilde{\boldsymbol{S}}^{\dagger}$ corresponds to the least-squares inverse of the matrix $\tilde{\boldsymbol{S}}$ and $\boldsymbol{S}_1^{\dagger}$, $\boldsymbol{S}_2^{\dagger}$ are the effective MAS algorithm inverse systems for each component.

The component periods are estimated using the double difference function (DDF) algorithm [1], i.e., by finding integers $\hat{N}_1$, $\hat{N}_2$ that minimize the following mean absolute error

$$\mathrm{DDF}[\hat{N}_1,\hat{N}_2] = \sum_{m=0}^{L-1} |x[n+m] - x[n+m+\hat{N}_1] - x[n+m+\hat{N}_2] + x[n+m+\hat{N}_1+\hat{N}_2]|, \tag{15}$$

where L is the duration and n is the origin of the analysis frame.

The MAS algorithm also implicitly uses the harmonic reassignment approach. This can be seen clearly by looking at the constraint system in the time-domain rewritten as matrix constraints

$$\boldsymbol{C}_1\boldsymbol{x}_1 = 0 \quad \text{or} \quad \boldsymbol{C}_2\boldsymbol{x}_2 = 0, \tag{16}$$

where $\{\boldsymbol{C}_i\}_{lm} = \delta[(m-l)_{\mathrm{mod}\,N_i}]$ and $\delta[n]$ is the discrete-time unit pulse function which has unit amplitude for $n=0$ and is zero elsewhere. These constraints together with Eq. (5) implicitly perform the harmonic reassignment task. This becomes evident after multiplying both sides of Eq. (16) by the DFT matrix $\boldsymbol{W}_R$, where $\{\boldsymbol{W}_R\}_{lm} = \exp(-\mathrm{j}2\pi lm/R)$. Using the structure of the matrix product $\boldsymbol{W}_R\boldsymbol{C}_i$ we have

$$\{\boldsymbol{W}_R\boldsymbol{C}_i\}_{lm} = \sum_{q=0}^{R-1}\exp\left(-\mathrm{j}\frac{2\pi lq}{R}\right)\delta[(m-q)_{\mathrm{mod}\,N_i}] = \exp\left(-\mathrm{j}\frac{2\pi lm}{R}\right). \tag{17}$$

The constraints on the $x_i[n]$ in the frequency domain can then written as

$$\boldsymbol{W}_R\boldsymbol{C}_i\,\boldsymbol{x}_i = \sum_{n=0}^{N_i-1}\exp\left(-\mathrm{j}\frac{2\pi nk}{R}\right)x_i[n] = X_i(\Omega_k) = 0. \tag{18}$$

So the constraints in the time-domain that force the dc value of the subsampled[4] versions of $x_i[n]$ to zero are equivalent in the frequency domain to forcing spectral nulls on $x_i[n]$ at $\Omega = \Omega_k$.

The spectral amplitude of the composite signal at $\Omega = \Omega_k$ can also be divided between the components by using the time-domain matrix constraints or their frequency-domain counterparts:

$$\boldsymbol{C}_1\boldsymbol{x}_1 = \lambda\boldsymbol{C}_2\boldsymbol{x}_2 \;\Leftrightarrow\; \boldsymbol{W}_R\boldsymbol{C}_1\boldsymbol{x}_1 = \lambda\boldsymbol{W}_R\boldsymbol{C}_2\boldsymbol{x}_2 \;\Leftrightarrow\; X_1(\Omega_k) = \lambda X_2(\Omega_k). \tag{19}$$

These constraints use a single parameter λ to distribute the spectral amplitude of the composite signal identically at $\Omega = \Omega_k$ corresponding to the option OP3.

The fourth option corresponds to the case where the constraints are applied on both components

$$\begin{pmatrix}\boldsymbol{C}_1 & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{C}_2\end{pmatrix}\begin{pmatrix}\boldsymbol{x}_1\\ \boldsymbol{x}_2\end{pmatrix} = 0 \;\Leftrightarrow\; X_1(\Omega_k) = X_2(\Omega_k) = X(\Omega_k) = 0. \tag{20}$$

corresponding to the option OP4 and in this case Eq. (5) will not be satisfied at $\Omega = \Omega_k$ unless the frequency content of the composite signal at these frequencies is already zero.

3.2. Restoration for comb filter separation

Component separation can also be accomplished via harmonic cancelation based FIR comb filters with impulse responses h_i and frequency responses H_i:

$$h_i[n] = \frac{1}{2}\{\delta[n] - \delta[n-N_i]\}, \quad i = 1,2,$$
$$H_i(\Omega) = \mathrm{j}\exp\left(-\mathrm{j}\frac{\Omega N_i}{2}\right)\sin\left(\frac{\Omega N_i}{2}\right), \tag{21}$$

[4] Downsampling the components by the factor R transforms them into components with periods corresponding to incommensurate fundamental frequencies.

where N_i the periodicity of the ith component has been exploited to eliminate its spectral content at multiples of the corresponding fundamental frequency. These comb filters only use the periodicity information regarding the components and do not exploit simultaneously the additive nature of the signal mixture. These filters eliminate the spectral content of both components at the common harmonics and therefore may violate the known information in Eq. (5). These filters also do not have the requisite frequency resolution to sufficiently attenuate the harmonics of the interfering component and accentuate the harmonics of the desired component when those harmonics are close and fall within the same cycle of the factor $\sin(\Omega N_i/2)$ in the frequency response.

If the periodicity information is used in conjunction with the additive nature of mixture, i.e.,

$$\begin{aligned} x_1[n] - x_1[n-N_1] = 0 &\Leftrightarrow H_1(\Omega)X_1(\Omega) = 0,\\ x_2[n] - x_2[n-N_2] = 0 &\Leftrightarrow H_2(\Omega)X_2(\Omega) = 0,\\ x_1[n] + x_2[n] = x[n] &\Leftrightarrow X_1(\Omega) + X_2(\Omega) = X(\Omega), \end{aligned} \tag{22}$$

we obtain the component separation system comprising of the two IIR filters:

$$\begin{aligned} T_1(\Omega) &= \frac{\zeta H_2(\Omega)}{H_1(\Omega) + \zeta H_2(\Omega)}, \quad \Omega \neq \Omega_k,\\ T_2(\Omega) &= \frac{H_1(\Omega)}{H_1(\Omega) + \zeta H_2(\Omega)}, \quad \Omega \neq \Omega_k,\ \zeta \in \mathbb{R}, \end{aligned} \tag{23}$$

with $R = \gcd(N_1, N_2)$. At $\Omega = \Omega_k$ the frequency responses via the L'Hospital's rule become

$$\begin{aligned} T_1(\Omega_k) &= \lim_{\Omega \to 2\pi k/R} T_1(\Omega) = \frac{\zeta N_2}{N_1 + \zeta N_2},\\ T_2(\Omega_k) &= \lim_{\Omega \to 2\pi k/R} T_2(\Omega) = \frac{N_1}{N_1 + \zeta N_2}. \end{aligned} \tag{24}$$

The spectral amplitude distribution ratio of the first to the second component at $\Omega = \Omega_k$ is $\lambda_k = \zeta N_2/N_1$ and corresponds to the inclusive option OP3 of the SPM problem. A parameter value of $\zeta = 0$ corresponds to the option OP1, when $\zeta = N_1/N_2$ the amplitudes are divided equally, and $\zeta = \infty$ corresponds to the option OP2. Note that $T_1(\Omega) + T_2(\Omega) = 1$ at all frequencies including $\Omega = \Omega_k$, thereby satisfying the known information in Eq. (5).

4. Experimental results

4.1. Experiments with sinusoidal signals

Consider the example of a mixture of two sinusoidal signals with carrier frequencies of $\Omega_{ci} = \pi/2, \pi/2.005$ and amplitudes of $A_{ci} = 2{,}4$. The exact periods of the components are $N_1 = 4$ and $N_2 = 401$ samples, respectively. The component period estimates from the DDF algorithm are $\hat{N}_1 = 392$ and $\hat{N}_2 = 401$ samples as shown in Fig. 1(b). Note that using the DDF period estimate $\hat{N}_1$ which is a multiple of the actual period N_1 does not affect the results of the MAS algorithm. The estimated periodicities are coprime and $\Omega = 0$ is the only common harmonic between the components. The spectral harmonics of the two components, however, are extremely close to each other.

The carrier frequencies of the components are extremely close. The comb filters with $\zeta = 0.587$ are implemented using FFTs of ten times the component period duration. These comb filters due to their frequency resolution problems are unable to effect simultaneous harmonic restoration and time-domain separation, thereby producing RMS separation errors of 24.8% and 6.49%. This is evident from both the time-domain estimates shown in Fig. 1(e,f) showing attenuation of the first component and the Fourier spectra in Fig. 1(g,h). The MAS algorithm, however, is free of these limitations which is evident from both time-domain estimates with $\lambda = 0$ shown in Fig. 1(c,d) and their Fourier spectra in Fig. 1(g,h). The corresponding RMS separation errors for the MAS algorithm are negligible.

4.2. Experiments with AM–FM signals

Consider the example with the sinusoidally modulated two-component AM–FM signal shown in Fig. 2(a) with parameters: $\Omega_{ci} = \pi/2$, $\pi/2.20$, $\Omega_{fi} = \pi/200, \pi/220$, 2, 4% FM, 1, 3% AM, $A_{ci} = 2$, 4, $\alpha_i = 0, \pi/2$, and $\beta_i = \pi, 0$. The actual periods of

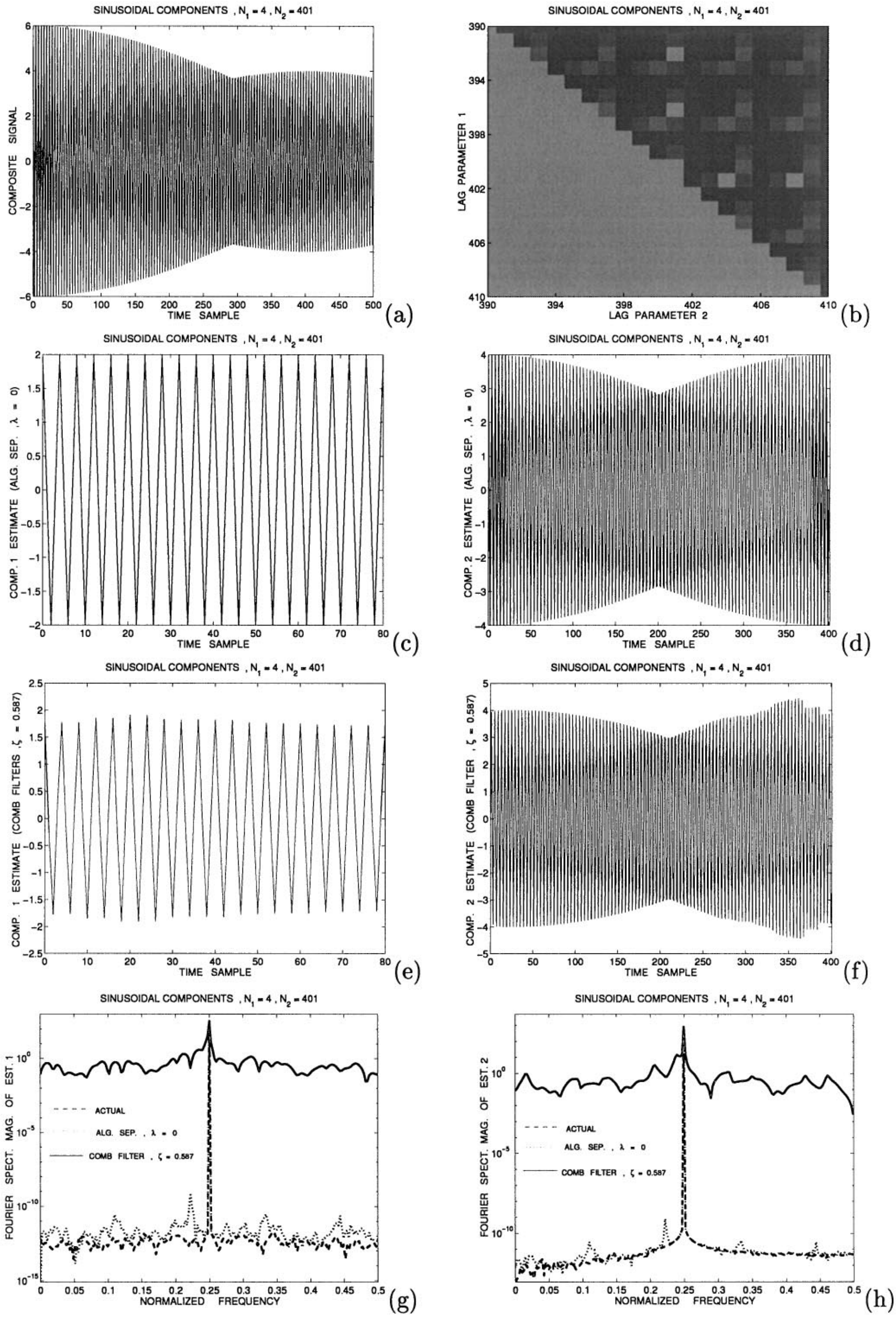

Fig. 1. Sinusoidal example. (a) Composite signal, (b) DDF over a half-quadrant search region, where dark areas indicate high DDF value and light areas indicate small DDF values, (c,d) MAS algorithm time-domain estimates using $\lambda = 0$, (e,f) comb filtering time-domain estimates using $\zeta = 0.587$, and (g,h) Fourier spectral magnitude of the estimates of the MAS algorithm and the comb filters.

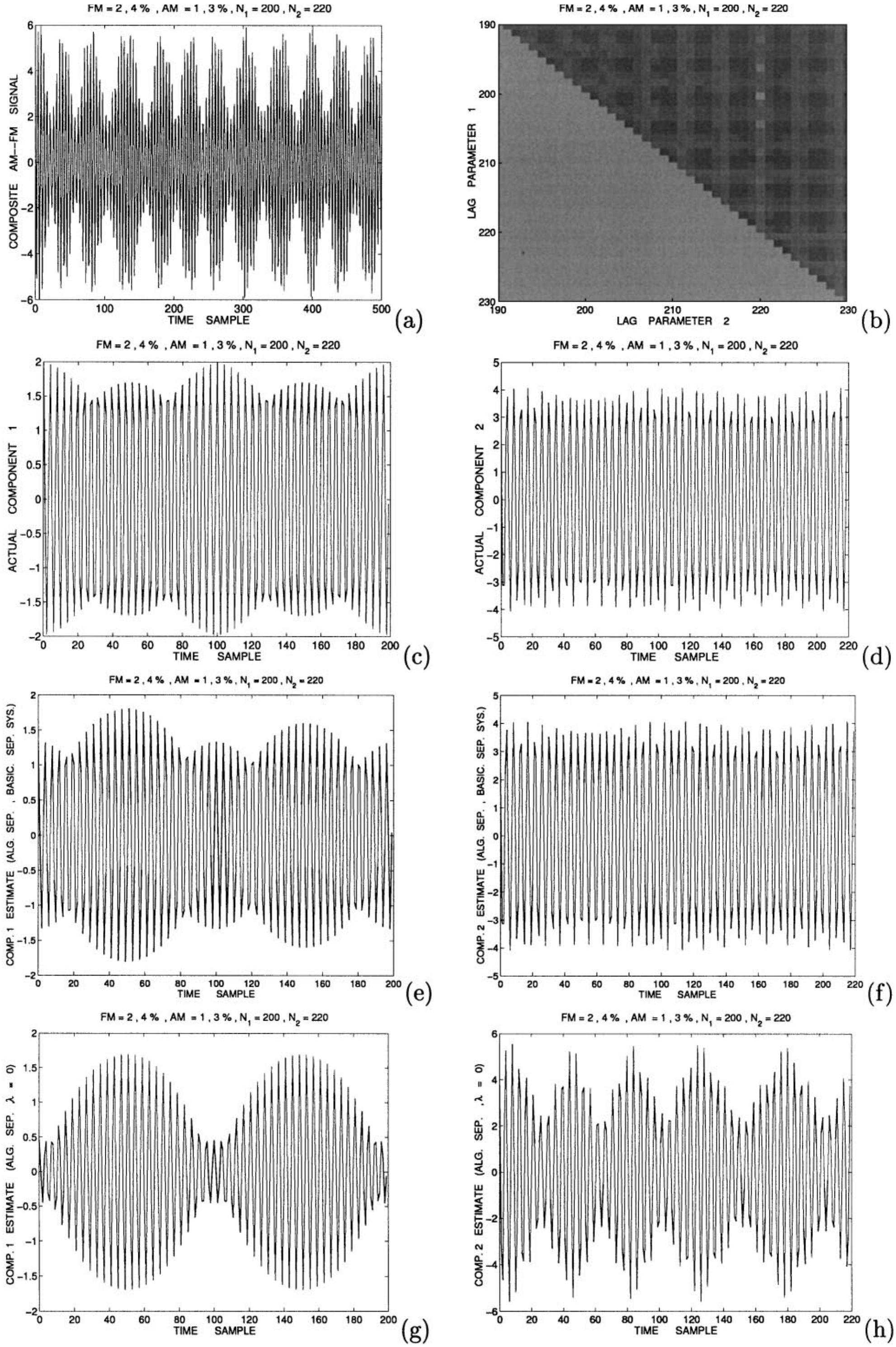

Fig. 2. Sinusoidal AM–FM signals. (a) Composite two-component AM–FM signal with sinusoidal FM and AM, (b) DDF over a half-quadrant search region, where dark areas indicate high DDF value and light areas indicate small DDF values, (c,d) actual time-domain signals, (e,f) MAS algorithm time-domain estimates via the SVD-inverse of the basic separation system, and (g,h) MAS algorithm time-domain estimates with $\lambda = 0$.

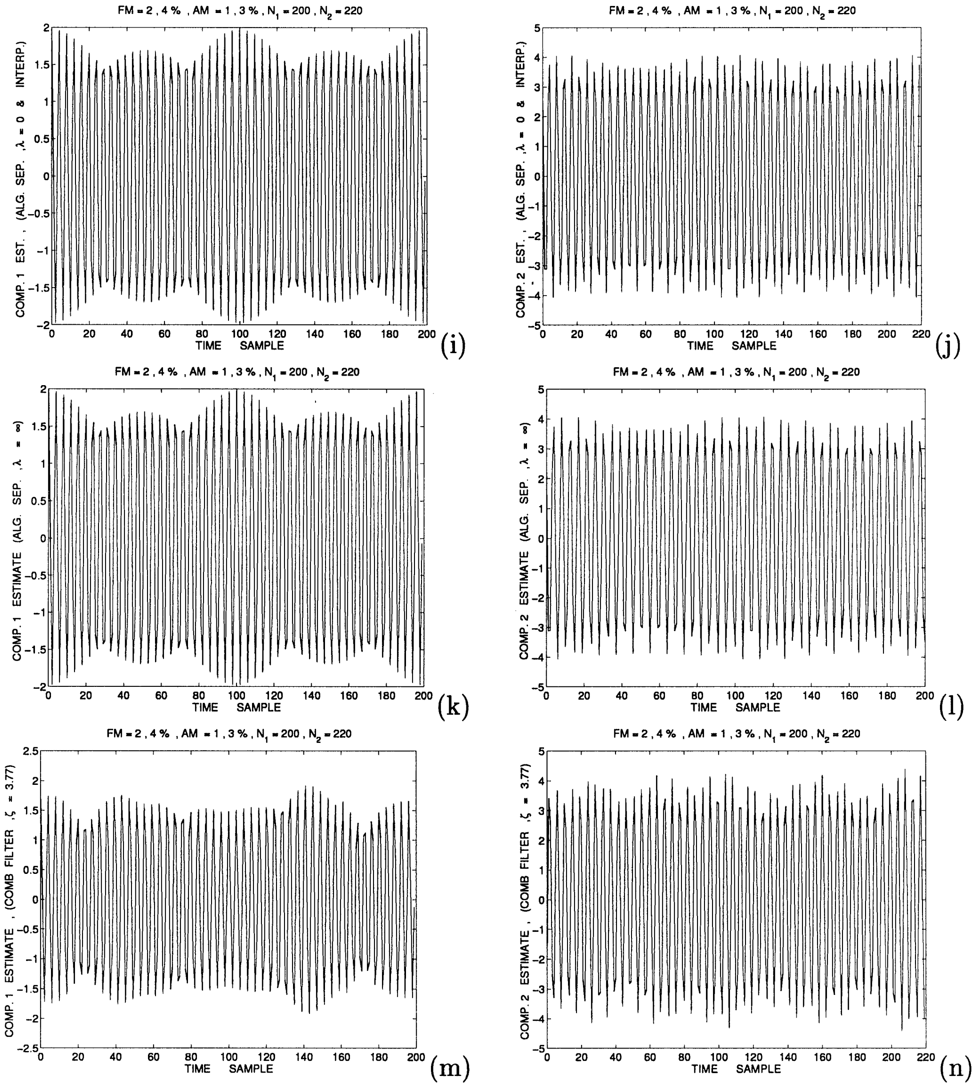

Fig. 2 (i, j) MAS algorithm time-domain estimates ($\lambda = 0$) after nearest-neighbor interpolation of the second component, (k,l) MAS algorithm time-domain estimates using the heuristic rule in Eq. (9) with $\lambda = \infty$, and (m,n) comb filter time-domain estimates with $\zeta = 3.77$.

the components are $N_1 = 200$ and $N_2 = 220$ samples, respectively. The component periods estimated using the DDF algorithm, as shown in Fig. 2(b), are $\hat{N}_1 = 200$ and $\hat{N}_2 = 220$ samples with $R = 20$ common harmonics.

First, we separate the components by inverting the rank deficient basic separation system via its SVD-inverse. Due to this lack or loss of information regarding the distribution of composite signal spectral information at the R common harmonics, the Fourier spectra of the estimates exhibit wide deviations from the actual spectra at these frequencies as evident from Fig. 2(q,r) resulting in large RMS separation errors of 36.38% and 18.19% that are evident from the time-domain estimates shown in Fig. 2(e,f). The actual time-domain signals are shown in Fig. 2(c,d).

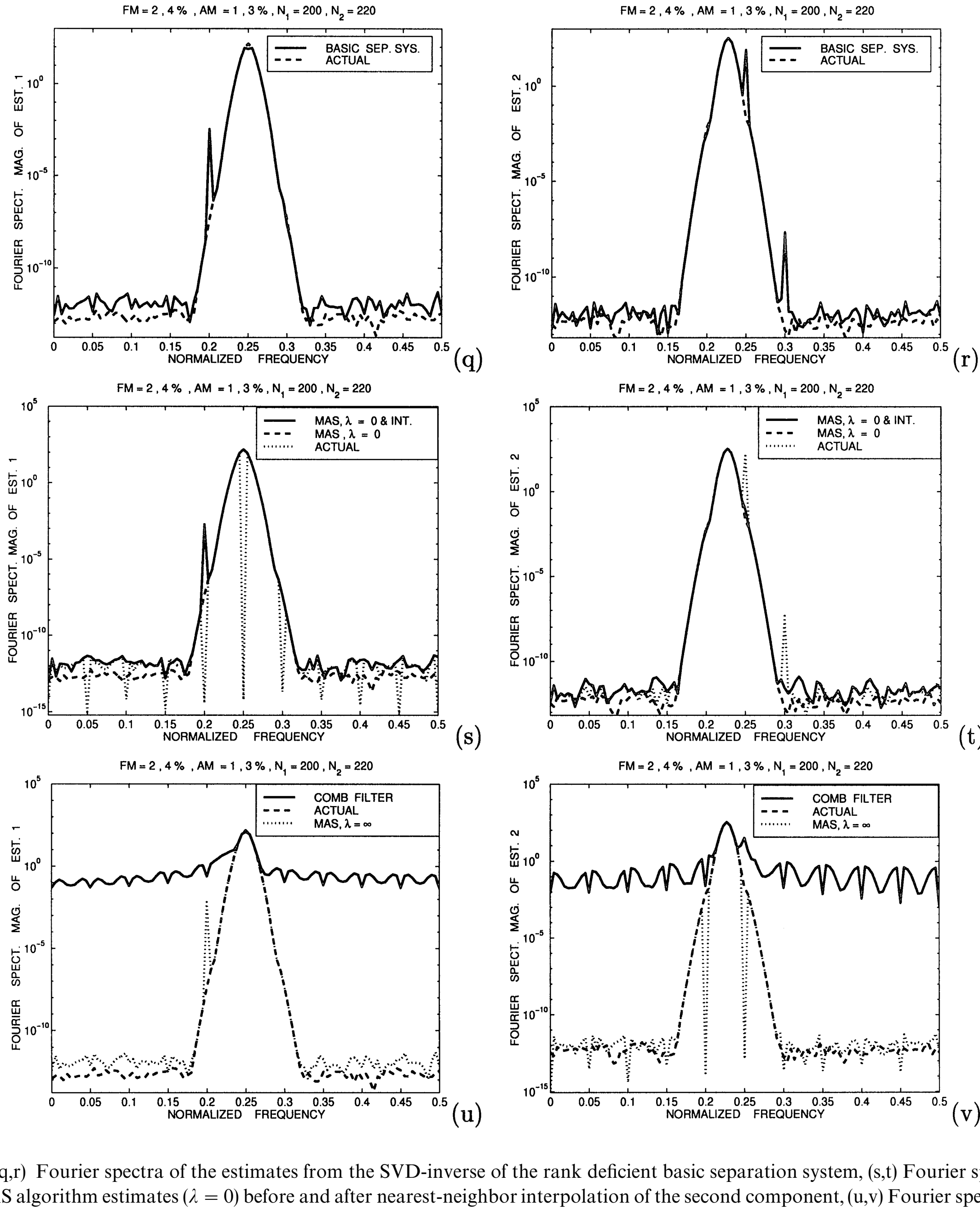

Fig. 2 (q,r) Fourier spectra of the estimates from the SVD-inverse of the rank deficient basic separation system, (s,t) Fourier spectra of the MAS algorithm estimates ($\lambda = 0$) before and after nearest-neighbor interpolation of the second component, (u,v) Fourier spectrum of the MAS algorithm using the heuristic rule with $\lambda = \infty$ and linear comb filtering.

Second, assuming no prior knowledge of the carrier frequencies we apply constraints on the first component, i.e., $\lambda = 0$. These constraints will force spectral nulls at the common harmonics on the first component. The common harmonic at $\Omega = \pi/2$ falls exactly at the first carrier frequency as shown in Fig. 2(s,t) producing very large RMS separation errors of 76.4% and 38.19% as evident from the time-domain estimates in Fig. 2(g,h). These forced spectral nulls produce attenuation of the first component time-domain estimate and amplification of the second component estimate.

Under the assumption of spectral correlation in the local neighborhood of the common harmonics, some of the lost spectral information can be restored via nearest-neighbor interpolation in conjunction with Eq. (9). From the estimated component periods, we determine the common harmonic locations and apply nearest-neighbor interpolation on the second component. Eq. (5) is then used to determine the spectral impulse amplitudes of the first component at the common harmonics as shown by the Fourier spectral magnitudes in Fig. 2(s,t) resulting in a reduction of the RMS separation error to 0.04% and 0.02% as described by the time-domain estimates in Fig. 2(i, j).

Using the heuristic rule in Eq. (9), however, the common harmonic at $\Omega = \pi/2$ is closer to first component so the constraints are applied on the second component, i.e., $\lambda = \infty$. This produces spectral nulls in the second component as shown in Fig. 2(u,v), while the MAS algorithm time-domain estimates are shown in Fig. 2(k,l) with very small RMS separation errors of 0.009% and 0.005%. The comb filters with $\zeta = 3.77$, however, are unable to effect satisfactory component separation. This is evident from both the Fourier spectral magnitude of the estimates in Fig. 2(u,v) resulting in large RMS separation errors of 17.2% and 7.98% and from the time-domain estimates shown in Fig. 2(m,n).

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